

MoMATUNE: MODEL, MATCH, THEN TUNE LANGUAGE MODELS FOR NUMERICAL RECORDS

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ABSTRACT

Despite remarkable progress in language understanding and generation, decoder language models continue to exhibit striking failures on numerical data. Numerical examples are typically serialized as text by concatenating feature names with their values (e.g., `fat:3.4`), while conventional task adaptation applies loss only to the downstream prediction target. This leaves two structures weakly supervised: *within-value sensitivity*, where digit position determines numerical magnitude, because token cross-entropy contains no explicit numerical metric; and *across-field structure*, where measurements in the same record are statistically dependent, because target-only tuning provides no direct loss on the measurements or their dependencies. We introduce MoMATUNE (**Model**, **Match**, then **Tune**), a two-stage post-training strategy organized around these supervision gaps. Stage 1 applies MODEL, ordinary next-token prediction densely over label-free serialized records, and optionally adds MATCH, a same-record consistency objective that aligns representations of different partial views of the same example. Stage 2 applies TUNE, supervised fine-tuning for downstream target prediction. We evaluate MoMATUNE on 169,285 records from eight public datasets, with 339 of 340 input fields numerical. At 1.5B parameters, MODEL raises the eight-dataset masked-value imputation aggregate from 0.106 to 0.401, while the three-dataset correct-context gain rises from 0.058 to 0.167. On diagnostics, MODEL also increases sensitivity to numerically consequential digit changes, while MATCH improves robustness to informative partial views. Adding TUNE yields higher target-supervised embedding aggregates for each tested Stage-1 parent, and the two-stage recipes obtain higher aggregate scores than TUNE alone. The same pattern is observed at 4B, 9.6B, and 27B parameters when model embeddings are used for downstream prediction.

1 INTRODUCTION

Many scientific and industrial examples contain little or no natural language; they consist primarily of numerical measurements. To process them with decoder-only LMs such as Qwen (Bai et al., 2023), a common approach is to serialize named measurements as attribute-value pairs, for example `<protein:6.2> <fat:3.4>`. We call each pair a *field* and the resulting sequence a *concatenated record*. Although this serialization makes numerical examples compatible with pretrained LMs, the numerical meaning of each value and the relationships among measurements remain implicit in the token sequence.

Prior work uses such serializations with task-specific supervision for prediction and generation (Hegselmann et al., 2023; Borisov et al., 2022; Wen et al., 2024; Wang & Fritz, 2024; Schindler et al., 2025). Complementary methods modify tokenization or training objectives to encode magnitude, order, or feature structure more explicitly (Golkar et al., 2023; Yan et al., 2024; Ucar et al., 2021; Zha et al., 2023). These directions demonstrate that LMs can operate on structured data, but leave two questions unresolved: *Can a standard causal decoder learn numerical significance within values and conditional relationships across fields?*

In this paper, we identify two structural requirements of numerical records that help explain current LM numeracy weaknesses and examine them with complementary diagnostics. First, *within-value*

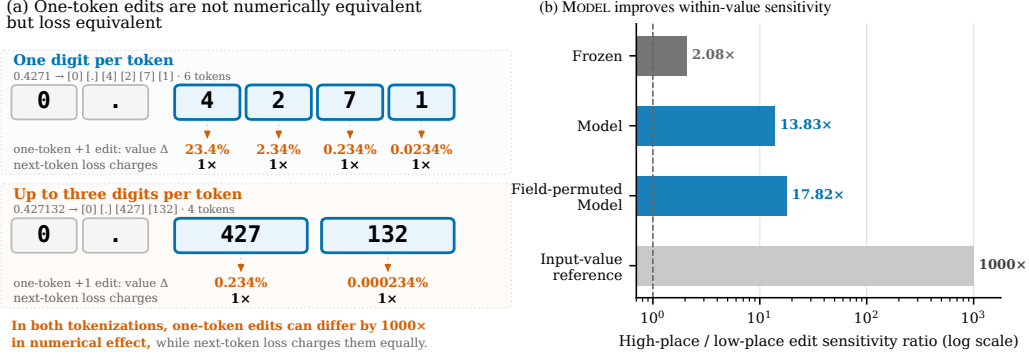


Figure 1: Token loss is numerically agnostic, but record-level adaptation improves magnitude sensitivity than target-only SFT. (a) With one digit per token, incrementing the first versus fourth fractional digit of 0.4271 produces a $1000\times$ difference in numerical effect. With groups of up to three digits, incrementing $[427]$ versus $[132]$ by one in 0.427132 also produces a $1000\times$ difference. Each intervention changes exactly one token. (b) We measure within-value sensitivity as the ratio between how far the pooled record embedding moves after a first-fractional-digit edit and after a fourth-fractional-digit edit. On the MiniBooNE diagnostic, applying the same objective densely through field-permuted MODEL increases this ratio from $2.08\times$ to $17.82\times$ at 1.5B. The $1000\times$ difference in numerical effect is a reference, not a required embedding-distance ratio. MODEL denotes next-token prediction on records, and field-permuted means reordering the attribute-value fields as data augmentation.

sensitivity requires distinguishing digit positions by their effect on numerical magnitude. Fig. 1(a) shows why digit position matters: in 0.4271 , increasing the first digit after the decimal changes the value by 23.4%, whereas increasing the fourth changes it by only 0.0234%, a $1000\times$ difference. Appx. B reports the tokenizer survey. Token cross-entropy is numerically agnostic: it provides no explicit weighting by numerical consequence, although dense record-level prediction can still learn such structure from the data distribution. In a MiniBooNE digit-perturbation diagnostic, the frozen 1.5B model is only $2.08\times$ more sensitive to the first-digit edit than to the fourth-digit edit (Fig. 1(b)). Second, *across-field structure* requires using relationships among measurements in the same record. If $\mathbf{x} = (x_1, \dots, x_d)$, predicting a missing field x_j requires modeling $p(x_j \mid \mathbf{x}_{-j})$, rather than only the marginal distribution of x_j . Across the eight datasets, the frozen 1.5B model obtains a masked-value imputation aggregate of 0.106, below the 0.212 field-conditional median baseline (Tab. 2). In a three-dataset context diagnostic, its correct-versus-mismatched context gain is only 0.058 (Tab. 3). Together, these complementary diagnostics show limited sensitivity to numerical magnitude in the digit-position test and limited use of cross-field context in masked-value completion.

These two challenges motivate three separable post-training roles: learn numerical and conditional record structure, optionally preserve useful information across partial views when inputs may be incomplete or reordered, and expose the resulting representation for target prediction. The second role is deployment-dependent rather than universally beneficial.

We therefore propose MOMATUNE, a modular two-stage post-training framework whose core path is MODEL $\rightarrow$ TUNE. Stage 1 applies MODEL, ordinary next-token prediction densely over every visible token of label-free numerical records, to strengthen within-value sensitivity, record completion, and use of across-field context. For settings that require robustness to partial records, Stage 1 can additionally include MATCH (same-record consistency), a regularizer that brings representations of alternative partial views of the same example closer together. We study both disjoint field subsets and *permuted-prefix views*, formed by independently permuting field order. Stage 2 applies TUNE (SFT): answer-masked supervised fine-tuning in a fixed field order for downstream target prediction. Thus, MODEL $\rightarrow$ TUNE forms the core training path, while MATCH is an optional robustness component whose effect depends on the chosen partial-view construction. One robustness-oriented variant MOMATUNE, uses field-permuted MODEL with MATCH, followed by fixed-order TUNE. MATCH draws on unified generative-representational training (Muennighoff et al., 2025), same-instance subset learning (Ucar et al., 2021), and representation consistency, but uses neither labels nor negative mining.

We evaluate MOMATUNE through frozen-embedding prediction and direct decoder tests across eight public datasets containing 169,285 records. The evaluated dataset is almost entirely numerical: 339 of 340 input fields are numerical, and seven of the eight datasets contain only numerical input fields. Our contributions are threefold:

- **Identify two numerical-record challenges.** We identify and empirically diagnose *within-value sensitivity* and *across-field structure* as two requirements that current decoder LMs weakly capture: models must represent how digit position determines magnitude and how measurements within the same record depend on one another.
- **Introduce a modular post-training strategy.** The core MODEL→TUNE path first adapts the decoder to numerical-record structure and then specializes the resulting representation for target prediction. MATCH is an optional same-record consistency regularizer for partial-view robustness whose utility depends on the paired views.
- **Evaluate numerical-record competence comprehensively.** We evaluate embedding and generation tasks on a near-purely numerical benchmark across multiple datasets and model scales, using matched controls to isolate the contribution of each training stage and data-view choice.

2 RELATED WORK

Numerical representation learning. Standard LM objectives do not explicitly encode numerical distance or place value, and numerical representations can remain weak on magnitude and arithmetic tests (Wallace et al., 2019; Deng et al., 2026). Prior work adds numerical structure through modified tokenization, feature-aware pretraining, subset views, or target-distance objectives (Golkar et al., 2023; Yan et al., 2024; Ucar et al., 2021; Zha et al., 2023). We instead ask whether a standard causal decoder, with unchanged tokenization and attention, can learn numerical magnitude and cross-field structure directly from label-free serialized records.

Joint generation and representation learning. GRIT jointly trains generation and representations (Muennighoff et al., 2025), while E5-Mistral, LLM2Vec, and NV-Embed develop decoder-derived encoders using supervised pairs, modified attention, specialized pooling, or embedding-focused training (Wang et al., 2024; BehnamGhader et al., 2024; Lee et al., 2025). Our goal is instead to improve numerical representations while preserving causal decoding, then specialize them for downstream prediction.

Serialized-record language modeling. TabLLM, GReaT, GTL, and TabuLa apply language models to serialized records for prediction or generation (Hegselmann et al., 2023; Borisov et al., 2022; Wen et al., 2024; Wang & Fritz, 2024). These works establish the usefulness of serialized records, but do not directly study what numerical structure a causal decoder can acquire from label-free record modeling before target supervision. Our work focuses on this question and evaluates how MODEL strengthens numerical-record representations before TUNE specializes them.

3 MOMATUNE: A POST-TRAINING FRAMEWORK FOR NUMERICAL RECORDS

3.1 DEFINITION: RECORDS, SERIALIZATION, AND EMBEDDINGS

A numerical record r is one dataset example containing m fields: $r = ((k_1, v_1), \dots, (k_m, v_m))$. Each field stores one measurement as a key–value pair: k_j names the measured variable, and v_j is its observed numerical value in this record. Serialization maps this ordered field sequence to text. Preserving the dataset’s column order, we define the serialized record as

$$x(r) = \langle k_1 : v_1 \rangle \cdots \langle k_m : v_m \rangle. \quad (1)$$

The downstream target y is excluded from Stage 1 and introduced only in Stage 2. For embedding evaluation, we represent the serialized record by the masked mean of final-layer hidden states,

$$z(x) = \frac{\sum_{t=1}^T a_t h_t}{\sum_{t=1}^T a_t}, \quad (2)$$

where a_t is the attention mask. Mean pooling is fixed across trained and frozen decoder arms.

One numerical record → three training signals

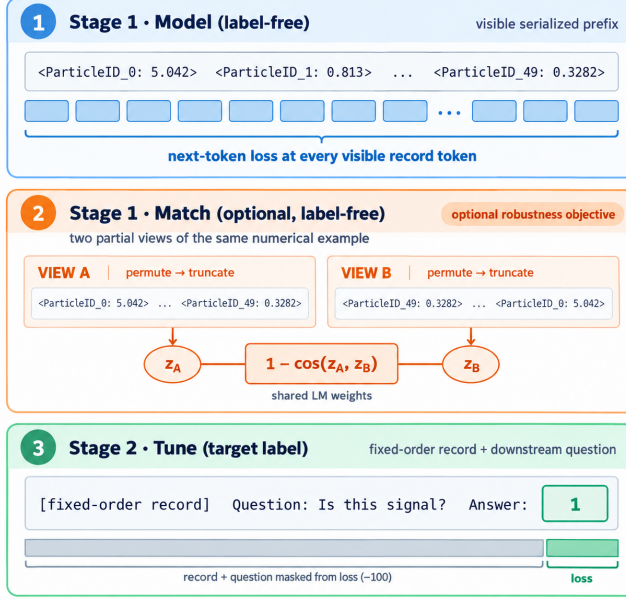


Figure 2: **One numerical record supports three available training signals.** MODEL applies next-token loss to every visible token in the truncated, label-free record. When partial-view robustness is desired, MATCH encodes two label-free permuted-prefix views with the same decoder and minimizes their cosine distance, without labels or negative pairs. TUNE returns to one fixed field order, masks the record and question tokens, and supervises only the label token.

Together, $x(r)$, $z(x)$, and y provide the common notation for the three training signals shown in Fig. 2. MODEL applies next-token loss to the visible tokens of $x(r)$; MATCH aligns the representations of two label-free views of the same record without observing y ; and TUNE appends a target question to $x(r)$, and supervises on y . Fig. 3 shows how these signals form the two-stage recipe; the following subsections define each stage and its objective.

3.2 STAGE 1: RECORD ADAPTATION

MODEL: next-token record adaptation. MODEL keeps ordinary causal next-token prediction but changes where it is applied: before target supervision, every visible token in a label-free record contributes to the loss:

$$\mathcal{L}_{\text{record}}(x) = - \sum_{t=1}^T \log p_{\theta}(x_t | x_{<t}). \quad (3)$$

This does not change the loss geometry; it supplies dense in-domain supervision for numerical values and their field context while preserving causal generation.

MATCH: conditional same-record consistency. We evaluate two constructions of positive views. In the fixed-order arms, the keys are randomly partitioned into a larger first view (approximately 50–75% of fields) and a disjoint second view; fields within each view remain in their original order. In the field-permuted arms, the field order is independently permuted twice and each serialization is then truncated at 512 tokens. We call these *permuted-prefix views*. They may overlap, but field-order permutation before truncation moves different key-value units into the visible prefix. Thus paired views generally differ in both order and visible content rather than preserving every value; Appx. D.3 reports the direct audit. After mean pooling and ℓ_2 normalization, both minimize

$$\mathcal{L}_{\text{cons}}(r) = 1 - \hat{z}(x(r_A))^{\top} \hat{z}(x(r_B)). \quad (4)$$

This asks two label-free views of the same example to have similar embeddings. There are no in-batch negatives, or variance regularizers. MODEL alone minimizes $\mathcal{L}_{\text{record}}$. When partial-view robustness is desired, the model-and-match arm instead uses

$$\mathcal{L}_{\text{Model+Match}} = \lambda_1 \mathcal{L}_{\text{record}} + \lambda_2 \mathcal{L}_{\text{cons}}. \quad (5)$$

The choice is therefore between a core MODEL checkpoint and a MODEL+MATCH checkpoint, rather than a requirement that every Stage-1 run use both losses. The consistency-only control drops $\mathcal{L}_{\text{record}}$ and directly tests whether MODEL prevents collapse. A MODEL+MATCH step uses three backbone forwards (visible record, view A, and view B), versus one for the MODEL baseline; therefore a null result cannot be attributed to giving the auxiliary arm less computation.

Field-order permutation: data augmentation. By default, each row is serialized once in the dataset’s original column order. This fixed presentation places every field in the same position and local context, limiting data diversity and encouraging positional shortcuts. Field-order permutation

MoMaTune: Model and Match numerical records, then Tune for the target

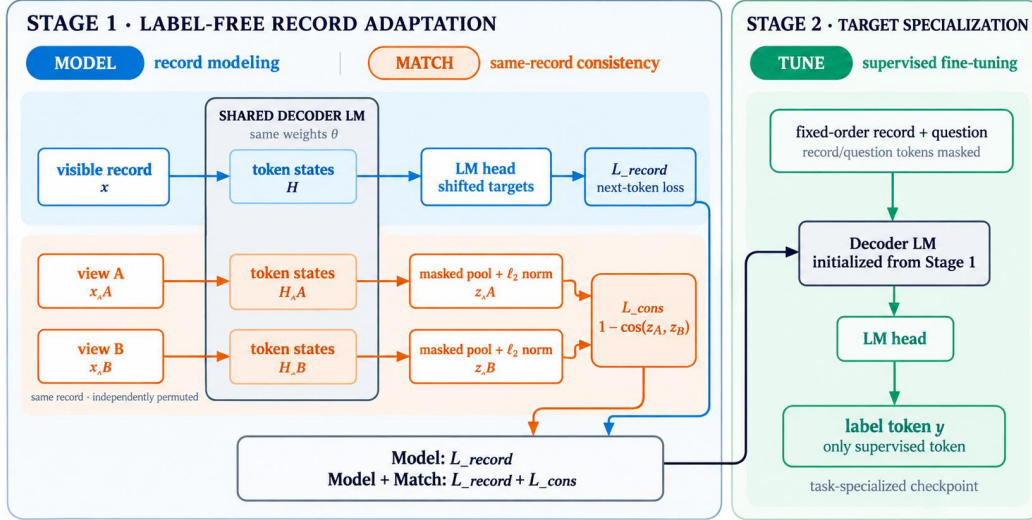


Figure 3: **MoMaTune** centers on **MODEL**→**TUNE**, with **MATCH** as an optional partial-view robustness component. Stage 1 always applies **MODEL** to label-free numerical records. Applications that require robustness to partial records can additionally apply **MATCH**. The resulting Stage-1 checkpoint initializes Stage 2, where **TUNE** presents a fixed-order record and target question and applies loss only to the label token.

addresses this by reordering complete key–value fields while preserving their names and values; multiple field-order permutations present the same measurements in varied contexts and encourage learning field identities and cross-field relationships without relying on a particular position.

3.3 STAGE 2: TARGET SPECIALIZATION

TUNE: supervised fine-tuning. Stage 1 is label-free and does not optimize for a downstream target. **TUNE** therefore specializes the Stage-1 checkpoint with SFT: each example concatenates $x(r)$, a target question, and answers y ; the record and question positions are masked, so only the answer tokens contribute to the loss. By default, **TUNE** uses one canonical field order and one view per record; field-permuted controls is not applied in this stage. Because **TUNE** observes y , its embedding score measures supervised target specialization rather than label-free competence.

4 EXPERIMENTAL DESIGN

4.1 EXPERIMENTAL SETTINGS

Datasets. We evaluate eight public datasets. Seven datasets are downloaded through OpenML (Vanschoren et al., 2014) and contain no text fields. They account for 161,389 of 169,285 records (95.3%). USDA FoodData is the only mixed-modality dataset: it adds one natural-language description to 48 numeric nutrients. Across the eight schemas, 339 of 340 input fields are numerical. All datasets use fixed 80/10/10 train/validation/test splits with seed 42. Appx. C summarizes the corpus in Tab. 6 and gives the complete construction and split statistics.

Models. Our primary experiments use Qwen2.5-1.5B-Instruct (Bai et al., 2023). To test whether the findings persist at larger scales, we additionally evaluate Qwen3-4B-Instruct-2507, Qwen3.5-9B (9.6B parameters), and Qwen3.8-27B (Yang et al., 2025). Because these checkpoints differ in architecture and tokenizer vocabulary, we treat this comparison as a model-size robustness sweep across checkpoint families. Appx. D.1 summarizes the model architectures and tokenizers; Appx. D.3 and D.2 provide the optimization settings and complete arm registry.

Evaluation. Our primary evaluation measures whether post-training makes record embeddings useful for downstream target prediction. At evaluation, for each model checkpoint, we get the frozen vector per record and fed to standardized linear probe: logistic regression evaluated by ROC–AUC on two

Table 1: Our MOMATUNE improves linear-probe target prediction across eight nearly pure-numerical datasets. Classification tasks report ROC-AUC, the regression tasks report R^2 . EMB-8 is unweighted mean of all eight dataset. The two field-permuted variants obtain the highest reported EMB-8 (0.753), with and without MATCH. All trained recipes in this table use a Qwen2.5-1.5B backbone; Tab. 4 reports the corresponding scale comparison from 1.5B to 27B. Additional classification metrics appear in Appx. Tab. 14, and regression metrics in Tabs. 15, 16, and 17.

Model / recipe	Classification: ROC-AUC $\uparrow$		Regression: R^2 $\uparrow$						Emb-8 $\uparrow$
	MiniBooNE	HIGGS	Supercond.	YearPred	Naval	CPU act.	Protein	USDA	
<i>Frozen decoder LMs (mean-pooled hidden states)</i>									
Qwen3-4B	.960	.642	.783	.160	.893	.926	.332	.940	.705
Qwen3-8B	.955	.636	.797	.172	.917	.931	.330	.946	.711
Qwen3.5-9B	.954	.644	.807	.149	.897	.928	.349	.939	.708
Qwen3-14B	.959	.660	.809	.156	.929	.947	.342	.951	.719
OLMo-2-7B	.961	.725	.817	.160	.930	.921	.329	.941	.723
DeepSeek-V2-Lite (16B MoE)	.958	.646	.778	.167	.837	.921	.334	.926	.696
gpt-oss-20b	.954	.668	.753	.153	.830	.803	.284	.915	.670
Gemma-4-12B	.957	.639	.813	.144	.921	.922	.353	.952	.713
Llama-3.1-8B	.960	.712	.804	.174	.915	.911	.321	.946	.718
<i>Embedding-specialized models (model-specific pooling)</i>									
gte-Qwen2-1.5B-instruct	.913	.569	.574	.014	.745	.954	.258	.670	.587
Qwen3-Embedding-4B	.943	.620	.690	.110	.716	.868	.257	.868	.634
Qwen3-Embedding-8B	.944	.620	.770	.118	.849	.893	.275	.903	.672
gte-Qwen2-7B-instruct	.923	.588	.642	.017	.795	.945	.325	.692	.616
LLM2Vec-Mistral-7B	.955	.619	.812	.162	.944	.902	.318	.904	.702
<i>MOMATUNE framework (Qwen2.5-1.5B backbone)</i>									
Zero-shot frozen decoder	.956	.655	.769	.152	.860	.891	.311	.910	.688
TUNE only	.963	.682	.773	.185	.884	.904	.380	.939	.714
MODEL $\rightarrow$ TUNE	.969	.691	.804	.198	.946	.934	.401	.943	.736
MODEL + MATCH $\rightarrow$ TUNE	.969	.695	.797	.211	.959	.949	.413	.950	.743
Field-permuted MODEL $\rightarrow$ TUNE	.963	.697	.817	.224	.960	.970	.424	.969	.753
Field-permuted MODEL + MATCH $\rightarrow$ TUNE (robustness variant)	.968	.691	.806	.219	.972	.965	.434	.967	.753

classification datasets and Ridge regression evaluated by R^2 on six regression datasets. Decoder LMs and trained recipes use mean-pooled final-layer hidden states; embedding-specialized models use their native pooling. As complementary direct-decoder evidence, masked-value imputation asks the model to recover a held-out field from the visible record; its primary metric is within-10% accuracy, summarized across datasets by GEN-8. Appx. C.2 defines every metric, protocol, and dataset-to-metric mapping.

4.2 EXPERIMENT RESULTS

Sec. 4.2.1 validates that our two-stage training framework obtains higher aggregate target-prediction scores than both the frozen backbone and TUNE alone. sec. 4.2.2 then separates the stages' roles: MODEL improves numerical sensitivity and record completion, MATCH improves incomplete-record robustness only when paired views preserve compatible evidence; and TUNE specializes the representation for target prediction. Sec. 4.3 tests whether the our two-stage framework gains persist from 1.5B to 27B, and other ablations.

4.2.1 OVERALL PERFORMANCE: TARGET PREDICTION AND NUMERICAL COMPLETION

Target prediction. Tab. 1 shows that applying MODEL before TUNE provides the central target-prediction increase. In the fixed-order setting, EMB-8 rises from 0.688 for the frozen checkpoint to 0.736 after MODEL $\rightarrow$ TUNE.¹ Adding MATCH to the fixed-order pipeline yields a higher EMB-8 of 0.743. Field-order permutation further raises the two-stage scores, from 0.736 to 0.753 without MATCH and from 0.743 to 0.753 with MATCH (Tab. 1; Appx. Tab. 11). Thus, the field-permuted MODEL $\rightarrow$ TUNE and field-permuted MODEL + MATCH $\rightarrow$ TUNE variants obtain the same aggregate EMB-8 of 0.753, showing that MATCH is not required for the highest complete-record aggregate in this comparison. Within the field-permuted pair, the variant without MATCH obtains higher scores on five of eight individual datasets (Appx. Tabs. 12 and 11). Together, these comparisons associate the main target-prediction increase with applying two-stage training, with field-order permutation providing an additional increase in the reported aggregate. We evaluate MATCH separately below for its conditional partial-view robustness effect.

Numerical completion. Tab. 2 evaluates numerical generation through masked-value completion. For each test example, we truncate the serialized input immediately after a selected `<key :` marker

¹The 1.5B recipes underperform the 7B frozen decoder on HIGGS and Superconductivity, but our 9.6B MODEL checkpoint obtains higher scores on both (Tab. 23).

Table 2: **Stage-1 training improves numerical completion.** We test the masked-value completion by asking the LM to generate the missing value. *Gen-8* is the unweighted mean across eight datasets. **MODEL** provides the largest improvement in numerical completion. Adding **MATCH** does not improve numerical completion, instead it benefits to robustness to partial views (Fig. 4(b)) and end-to-end target prediction (Tab. 1).

Model / recipe	Within-10% completion accuracy $\uparrow$								Gen-8 $\uparrow$
	MiniBooNE	HIGGS	Supercond.	YearPred	Naval	CPU act.	Protein	USDA	
<i>Zero-shot decoder LMs</i>									
Qwen3-4B	.006	.130	.292	.024	.144	.194	.000	.260	.131
Qwen3-8B	.004	.142	.282	.024	.152	.168	.000	.258	.129
Qwen3.5-9B	.006	.172	.280	.034	.122	.174	.000	.286	.134
Qwen3-14B	.002	.158	.294	.030	.140	.172	.000	.290	.136
OLMo-2-7B	.002	.124	.230	.006	.130	.202	.000	.194	.111
DeepSeek-V2-Lite (16B MoE)	.004	.132	.246	.024	.156	.148	.000	.220	.116
gpt-oss-20b	.008	.128	.128	.010	.008	.196	.000	.144	.078
Gemma-4-12B	.004	.132	.262	.028	.184	.176	.002	.284	.134
Llama-3.1-8B	.002	.106	.284	.008	.098	.212	.000	.258	.121
<i>MoMATUNE framework (Qwen2.5-1.5B backbone)</i>									
Zero-shot frozen decoder	.004	.126	.250	.028	.086	.158	.000	.196	.106
Field-permuted MODEL	.318	.292	.690	.062	.904	.364	.592	.358	.448
Field-permuted MODEL + MATCH	.306	.286	.676	.062	.912	.382	.582	.356	.445

and ask the decoder to generate the corresponding value. A prediction is correct when the parsed value is within 10% of the held-out value. We use only Stage-1 checkpoints instead of Stage-2 because the task measures general numerical completion before downstream target specialization. Our Stage-1 raises raises mean masked-value completion from 0.106 to 0.448. Adding **MATCH** does not further improve numerical completion: Gen-8 slightly reduces to 0.445 with field-order permutation. This is consistent with its different objective: same-record consistency favors representations stable across partial views (Appx. Tab. 20), whereas exact completion requires retaining view-specific numerical details. **MATCH**’s benefit instead appears in robustness to partial views (Fig. 4(b)). Overall, Tab. 2; combined with the magnitude-sensitivity and cross-field-context analyses in Sec. 4.2.2, they provide behavioral evidence that our Stage-1 improves numerical understanding.

4.2.2 MODEL, MATCH, AND TUNE CONTRIBUTE DISTINCT CAPABILITIES

MODEL increases reliance on the correct cross-field context. Tab. 3 evaluates whether the model can correctly do the masked-value completion with correct and mismatched records on three dataset (USDA, Superconductivity, and MiniBooNE). The *correct records* contains the correct values from the same example, while the *mismatched records* contains a mismatched value that preserves the field names, order, and target but replaces with values from another example. The resulting context gain, correct minus mismatched accuracy Δ_{ctx} , rises from 0.058 for the frozen model to 0.167 after fixed-order MODEL (Appx. C.2), and to 0.271 with field-permuted MODEL. For field-permuted MODEL, paired record-bootstrap intervals are positive on USDA (0.244, 95% CI [0.201, 0.289]), Superconductivity (0.472, [0.423, 0.520]), and MiniBooNE (0.096, [0.051, 0.139]). The context gain Δ_{ctx} is positive on all three datasets for both MODEL variants. Thus, across these three diagnostics, MODEL makes masked-value prediction depend more strongly on values from the correct cross-field context, indicating that the model has learned relationships among fields.

MODEL increases sensitivity to numerical magnitude. Changing a leading digit typically alters a number much more than changing a later digit, so a numerically sensitive representation should respond more strongly to the former. We test this by comparing the embedding changes caused by first- and fourth-digit edits. The sensitivity ratio rises from $2.08\times$ for the frozen model to $13.83\times$ with MODEL and $17.82\times$ with field-permuted MODEL (Fig. 1(b); Appx. Fig. 6). Relative to the frozen model, MODEL increases the ratio by 11.75 (95% CI [7.04, 17.09]) under a paired record bootstrap. Field permutation adds 3.99 relative to MODEL, but its interval crosses zero (95% CI [−3.91, 13.17]). Thus, MODEL increases sensitivity to numerically consequential digit changes, while this diagnostic does not resolve an additional effect of field permutation.

MATCH improves robustness to missing fields. Missing fields can shift a record representation and degrade its prediction. We test whether **MATCH** reduces this sensitivity by removing half of the input record fields and evaluating prediction from the same incomplete views with and

Table 3: **MODEL improves the utilization of cross-field context.** For each masked field, the correct and mismatched prompts have identical field names, order, target, and scorer; only the values in mismatched prompts are replaced by values from another record. Δ_{ctx} is correct minus mismatched within-10% accuracy. The first three columns are macro averages over three datasets; the remaining columns show the corresponding per-dataset context gains Δ_{ctx} .

Model / recipe	Macro within-10% accuracy			Per-dataset Δ_{ctx} $\uparrow$		
	Correct $\uparrow$	Mismatched	Δ_{ctx} $\uparrow$	USDA	Supercond.	MiniBooNE
Frozen base	.142	.084	+.058	+.064	+.115	-.004
MODEL	.338	.171	+.167	+.170	+.284	+.047
Field-permuted MODEL	.463	.193	+.271	+.244	+.472	+.096

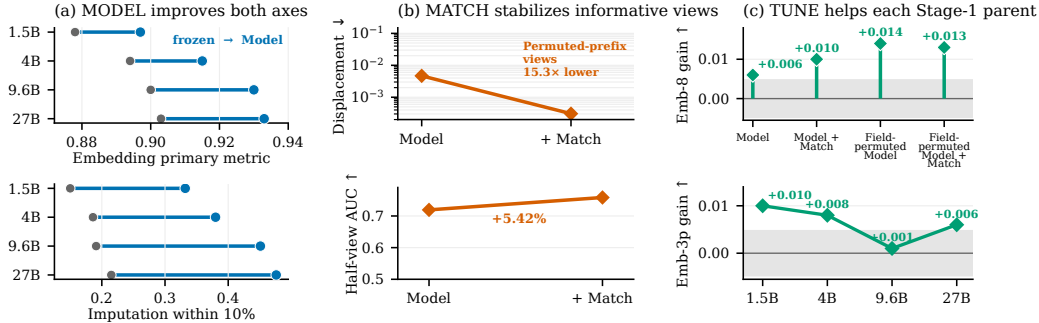


Figure 4: **MODEL and TUNE improve their intended capabilities, while MATCH stabilizes informative partial views.** (a) Across three-dataset, fixed-order MODEL improves both the primary target-prediction metric and numerical completion relative to the frozen checkpoint at every scale. (b) For permuted-prefix views, MATCH reduces embedding displacement by 15.3 $\times$ and raises half-view AUC by 5.4%. (c) The upper panel shows that TUNE improves EMB-8 for every tested Stage-1 parent at 1.5B; the lower panel shows a positive three-dataset gain through 27B for the field-permuted MODEL+MATCH parent.

without MATCH. When added to field-permuted MODEL, MATCH reduces the full-to-incomplete representation displacement from 0.00468 to 0.000306 (15.3 $\times$, 95% CI [15.257 $\times$, 15.354 $\times$]) and raises mean incomplete-view ROC-AUC by 5.4% from 0.719 to 0.758 (95% CI [0.023, 0.056]) (Fig. 4(b); Appx. Fig. 7 and Tab. 20). Thus, when both views retain useful target information, MATCH improves prediction under missing fields rather than complete-record accuracy.

MATCH can hurt when paired views contain complementary predictive information. For disjoint-half views, adding MATCH to fixed-order MODEL reduces representation displacement from 0.0916 to 0.0120, but lowers mean half-view AUC from 0.584 to 0.533 (Appx. Tab. 20). Each half may contain predictive fields absent from the other, so enforcing consistency can suppress view-specific but task-relevant information. MATCH is therefore most appropriate when paired views preserve compatible predictive evidence.

TUNE uses labeled examples to improve target prediction. In Tab. 1, at 1.5B, adding TUNE raises EMB-8 by 0.8–1.9% relative to its Stage-1 parent: fixed-order MODEL, fixed-order MODEL+MATCH, field-permuted MODEL, and field-permuted MODEL+MATCH (Fig. 4(c); Appx. Tab. 12). In Tab. 4, compared with TUNE alone, our MOMATUNE improves the target-prediction score at every tested scale from 1.5B to 27B, with relative gains of +2.6%, +2.2%, +1.0%, and +1.6%, respectively. In contrast, masked-value imputation decreases for every tested parent. Thus, labeled TUNE examples specialize the Stage-1 representation for target prediction while giving up some general record-completion ability.

Field-order permutation helps MODEL learn beyond fixed field positions. At 1.5B, field-order permutation raises the digit-sensitivity ratio from 13.83 $\times$ to 17.82 $\times$ (Fig. 1(b)), target prediction emb-8 increase from 0.743 to 0.753 (Tab. 1), masked-value imputation from 0.401 to 0.448 (Tabs. 2), and context gain from 0.167 to 0.271 when comparing fixed-order with field-permuted MODEL (Tab. 3). The equal-update 27B shares the same pattern: field-permuted MODEL exceeds the fixed-order control on every primary dataset and raises mean imputation from 0.479 to 0.613 and the three-dataset target-prediction mean from 0.920 to 0.936 (Fig. 5). Together, these comparisons

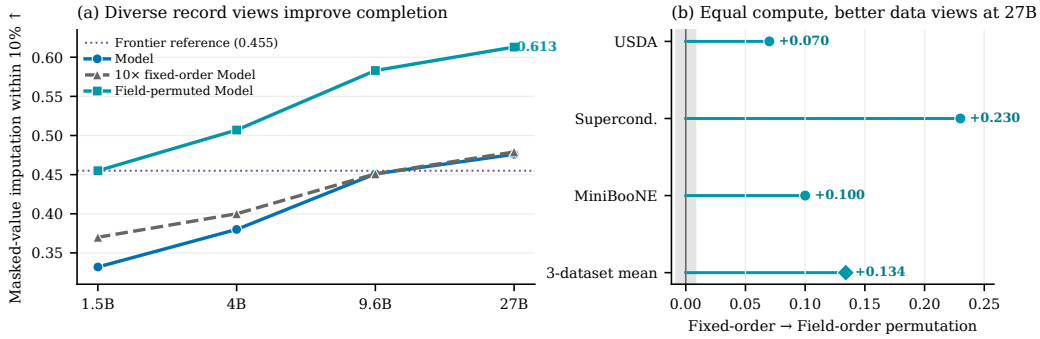


Figure 5: **Field-order permutation improves completion beyond an equal-update fixed-order control.** (a) Mean within-10% masked-value imputation over three-dataset from 1.5B to 27B, comparing standard fixed-order MODEL, extended fixed-order training, and field-permuted MODEL; the dotted line marks the 0.455 frontier-model reference used in this diagnostic. (b) At 27B, field-permuted MODEL improves every dataset over the equal-update fixed-order control, increasing the three-dataset mean by 0.134; the gray band marks the ± 0.008 same-configuration imputation spread.

Table 4: **Target-prediction comparisons across the tested model sizes.** Each cell reports before→after and the absolute difference in parentheses for the mean of MiniBooNE ROC-AUC and Superconductivity and USDA R^2 . All three comparisons show positive point differences at the four tested model sizes.

Scale	Frozen checkpoint → MODEL	Field-permuted MODEL+MATCH → MOMATUNE	TUNE only → MOMATUNE
1.5B	.878→.897 (+.019)	.904→.914 (+.010)	.891→.914 (+.023)
4B	.894→.915 (+.021)	.922→.930 (+.008)	.910→.930 (+.020)
9.6B	.900→.930 (+.030)	.934→.935 (+.001)	.926→.935 (+.009)
27B	.903→.933 (+.030)	.936→.942 (+.006)	.927→.942 (+.015)

support field-order permutation as a data augmentation that improves numerical sensitivity, target prediction, completion, and context use.

4.3 ABLATION STUDIES

Our two-stage training framework gains persist from model size 1.5B to 27B. Tab. 4 compares target-prediction scores across the 1.5B, 4B, 9.6B, and 27B checkpoints. Fixed-order MODEL consistently outperform its corresponding frozen checkpoint (2.2–3.3%), while the field-permuted MODEL+MATCH→TUNE variant outperforms TUNE alone (1.0–2.6%) across all four sizes. Adding TUNE to MODEL+MATCH also yields consistent gains (0.1–1.1%). Appx. Fig. 8 and Tab. 25 report masked-value imputation results, and Appx. Tab. 26 provides the complete 27B recipe grid.

Generality to text and categorical fields. Appx. E.3 shows that our two-stage training framework also generalizes to mixed text and numerical data. Evaluations on two non-numerical datasets (up to 95.3% non-numeric) show trends consistent with those in Tab. 1.

Sensitivity to loss weights. Appx. E.1.1 varies the weighting in Eq. 5. The default MODEL–MATCH weighting achieves the highest R^2 , and all tested $\lambda_2 \in [0.5, 2.0]$ settings are within 0.005.

5 CONCLUSION

Numerical records require language models to represent both magnitude within values and relationships across fields. Our results support a hierarchical division of labor in MOMATUNE. MODEL is the foundation: it adapts the decoder to numerical-record structure and improves both target prediction and numerical completion. TUNE then specializes the resulting representation for downstream target prediction, while reducing general record-completion ability. MATCH is optional: it can stabilize informative partial views, but it does not increase complete-record performance in the strongest field-permuted comparison and can reduce prediction quality when paired views contain complementary information. We therefore recommend MODEL→TUNE as the core path, with MATCH added only when partial-view robustness is required and the intended view construction has been validated.

AI USE STATEMENT

Generative AI tools were used for literature organization, code assistance, experiment bookkeeping, and polishing this manuscript. All citations were checked against source records; all numerical results were produced by executed code and checked against stored evaluation artifacts and logs. The authors reviewed all AI-assisted work and take responsibility for the final text, claims, code, and results.

ETHICS STATEMENT

This study presents minimal direct ethical risk. It uses public scientific and numerical benchmark datasets, involves no human participants or intervention, and does not collect personal or sensitive data. All evaluations are conducted offline for research purposes. Any future use in consequential applications would require separate domain-specific validation and oversight.

REPRODUCIBILITY STATEMENT

Appx. C documents dataset construction, splits, serialization, and target exclusion, while Appx. C.2 defines each evaluation task, metric, baseline, and aggregate. Appx. D.1, D.2, and D.3 specify the checkpoint families, complete recipe registry, optimization settings, seeds, and view construction. Full per-dataset results and matched controls appear in Appx. E; mechanism and failure-analysis protocols appear in Appx. F; and complete model-size results appear in Appx. G. Appx. H records the direct-decoder prompts, token limits, parsing rules, and excluded invalid cases. The supplementary material includes the result files and scripts used to regenerate the reported tables.

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A LIMITATIONS

The model-size sweep changes architecture and tokenizer vocabulary and these experiments therefore do not establish a fitted scaling law. Appx. E.3 does extend the full recipe block to two datasets whose records are largely categorical, including one in which 95.3% of the value characters are non-numeric, and there no recipe falls below the frozen baseline; that dataset is however close to a frozen probe’s ceiling, and the main results remain almost entirely numerical, so this evidence bounds the mixed-type question rather than settling it. Finally, TUNE directly observes the target while reducing imputation, so our conclusions are limited to post-training numerical records and target specialization rather than broad tabular or label-free generality.

B INTUITION: LLM NUMERACY ISSUE

A decimal reaches the model as several tokens, and current models split it in one of two ways. Under both conventions, changing a single digit changes exactly one token, so a one-hot training target charges the same price whether that digit was worth 23.4% or 0.0234% of the value.

- **One digit per token.** Every digit becomes its own token, so `0.4271` is six tokens, `(0) (.) (4) (2) (7) (1)`; used by Qwen2.5/3/3.5, Mistral, Gemma-4, DeepSeek-V2 and Llama-2.
- **Groups of up to three digits.** Digits are merged into chunks. In Fig. 1(a), `0.427132` is displayed as four tokens, `(0) (.) (427) (132)`. This convention is used by GPT-3.5 and GPT-4 (`cl100k_base`), GPT-4o and the o-series (`o200k_base`), Llama-3, OLMo-2 and phi-4.

We call a model *digit-blind* when its record embedding moves by about the same amount regardless of which digit was edited. We measure this by editing either the first or the fourth significant digit. Then compute $\bar{d}_1/\bar{d}_4$: how far the embedding moves when breaking the most important digit ($\bar{d}_1$), divided by how far it moves when breaking the least important one ($\bar{d}_4$). For example, number `0.4271`, editing the first digit gives `0.5271` and moves the value by 23.4%, while editing the fourth gives `0.4272` and moves it by 0.0234%; each edit replaces exactly one token. The $1000\times$ contrast is only a numerical input-scale reference. The diagnostic asks whether first-digit edits produce stronger embedding separation than fourth-digit edits, not whether the embedding-distance ratio reaches $1000\times$. Tab. 5 reports this for seven frozen pretrained models spanning both conventions, using the protocol of Appx. F.

Across the seven surveyed models, ratios lie between 1.12 and 2.14 against an input contrast of $1000\times$, and the two conventions overlap completely. On this diagnostic, digit-blindness therefore appears across the surveyed pretrained decoders rather than being isolated to one tokenizer or one model family, which motivates addressing it through training rather than re-tokenizing.

Table 5: Digit-blindness in frozen pretrained models, under both tokenization conventions. Each model is evaluated with its own tokenizer, so the accepted record set is derived per model and the sets are not identical across rows; the Qwen2.5-1.5B row is therefore an independent re-derivation of the frozen row of Tab. 19 (2.14 here versus 2.08 there). Larger ratios indicate greater leading-digit sensitivity under this proxy. The reference row contrasts a first-digit edit (0.4271→0.5271, a 23.4% change in value) with a fourth-digit edit (0.4271→0.4272, 0.0234%), a ratio of 1000×; it is a reference contrast, not an expected embedding ratio. MiniBooNE, 200 accepted records per model, frozen checkpoints only.

Tokenization	Model	$\bar{d}_1/\bar{d}_4 \uparrow$	Spearman $\rho \uparrow$
One digit per token	Qwen2.5-1.5B-Instruct	2.14	0.276
One digit per token	Qwen3-14B	2.04	0.196
One digit per token	Mistral-7B-v0.3	1.65	0.191
One digit per token	Gemma-4-12B	1.12	0.131
Groups of up to three	phi-4	1.94	0.353
Groups of up to three	OLMo-2-7B	1.81	0.159
Groups of up to three	Llama-3.1-8B	1.51	0.238
–	Underlying value change (e.g. 0.4271→0.5271)	1000.00	–

C DATASET AND EVALUATION METRICS DETAILS

C.1 DATASET CONSTRUCTION

Each OpenML dataset is converted to a float feature matrix, capped at 30,000 records where necessary, randomly permuted with NumPy seed 42, and split 80/10/10. Values are formatted with four significant digits; integer-valued floats are written as integers. Keys remain in the metadata column order. The materialized split is reused by every checkpoint, probe, and generation test.

The benchmark is deliberately numerical. MiniBooNE, HIGGS, Superconductivity, CPU activity, Naval propulsion, Protein structure, and YearPredictionMSD contain numerical input fields only. USDA FoodData contains 48 numerical nutrient fields and one natural-language food description. Thus 339 of 340 schema fields are numerical, and 95.3% of records come from numerical-only datasets. We do not synthesize descriptions or convert numbers into explanatory prose.

Table 6: Public evaluation corpus. Seven of eight datasets are numerical-only; across schemas, 339/340 input fields are numerical. “Records” includes train, validation, and test splits.

Dataset	Task	Numeric fields	Text fields	Records	Target
MiniBooNE	cls.	50	0	30,000	signal
HIGGS	cls.	28	0	30,000	class
Superconductivity	reg.	79	0	21,263	critical temperature
USDA FoodData	reg.	48	1	7,896	energy (kcal)
CPU activity	reg.	21	0	8,192	user CPU %
Naval propulsion	reg.	14	0	11,934	compressor decay
Protein structure	reg.	9	0	30,000	RMSD
YearPredictionMSD	reg.	90	0	30,000	year
Total		339	1	169,285	7/8 numeric-only

The downstream target is stored separately and is absent from every Stage-1 record-adaptation serialization. TUNE adds the target only through a question–answer suffix, as illustrated in Fig. 2. Under field-order permutation, the values and field set stay fixed; only the order of <key: value> units changes. After truncation to the model’s token budget, however, different field orders can expose different visible field subsets; this partial-view diversity is the intervention tested in the ablation. Tab. 7 gives all split sizes, target ranges, and generation sample counts.

Table 7: Complete dataset statistics and input modality. “Gen. n ” is the number of test records used for masked-value imputation and field lookup for each checkpoint.

Dataset	Numeric	Text	Train	Val.	Test	Gen. n	Target range / classes
MiniBooNE	50	0	24,000	3,000	3,000	500	binary
HIGGS	28	0	24,000	3,000	3,000	500	binary
Superconductivity	79	0	17,010	2,126	2,127	500	[0.00021, 185]
USDA FoodData	48	1	6,316	789	791	500	[0, 902]
CPU activity	21	0	6,553	819	820	500	[0, 99]
Naval propulsion	14	0	9,547	1,193	1,194	500	[0.95, 1.0]
Protein structure	9	0	24,000	3,000	3,000	500	[0, 20.999]
YearPredictionMSD	90	0	24,000	3,000	3,000	500	[1922, 2010]

C.2 EVALUATION DIMENSIONS AND METRICS

We organize evaluation along two complementary dimensions. The *embedding axis* measures what target information is linearly recoverable from frozen record representations. The *generation axis* measures whether the decoder can directly produce or score numerical record content without a fitted readout. Geometry diagnostics explain changes in representation space and are not folded into any headline aggregate.

Embedding axis. The embedding axis evaluates each trained checkpoint through post-hoc supervised probes. After training for an arm is complete, we freeze the language model and extract one representation from each record with the target omitted. We then use scikit-learn to fit a separate linear readout on the task’s training split: logistic regression for classification and Ridge regression for regression. We select regularization on validation, refit the selected probe on train+validation, and score it once on test. A *probe prediction* is the output of this readout; the probe neither updates the decoder nor participates in continued pretraining or TUNE.

$$\begin{aligned}\tilde{z} &= \text{Standardize}_{\text{train}}(z_{\theta}(x)), \\ p_{\phi}(y = 1 \mid x) &= \sigma(w_c^{\top} \tilde{z} + b_c), \quad \hat{y}_{\psi}(x) = w_r^{\top} \tilde{z} + b_r.\end{aligned}\tag{6}$$

Here $z_{\theta}(x)$ is the record representation produced by the frozen trained checkpoint, $\phi = (w_c, b_c)$ is the logistic-regression probe, and $\psi = (w_r, b_r)$ is the Ridge-regression probe. These are separate scikit-learn models, not an MLP or an added LM layer.

In all examples below, italic labels describe the evaluation pipeline and are not part of the model input. Monospaced lines show the input text; $\dots$ marks fields omitted only for display. Scoring references are labelled separately: they are absent from prediction and imputation inputs, whereas field lookup deliberately leaves the requested value in the record. Because predictions vary by checkpoint, the examples state the returned object or parsed prediction rather than substituting the reference value as if it were an observed model output.

- **Classification probe.** For MiniBooNE and HIGGS, logistic regression maps the frozen vector to a class probability. We choose $C \in \{0.01, 0.1, 1, 10\}$ by validation ROC-AUC. Test metrics are accuracy $N^{-1} \sum_i \mathbf{1}[\hat{y}_i = y_i]$ and ROC-AUC, which measures whether positive examples receive higher scores than negative examples across thresholds.
- **Regression probe.** For the six continuous-target datasets, Ridge regression maps the frozen vector to a scalar. We choose $\alpha \in \{0.1, 1, 10, 100, 1000\}$ by validation R^2 . Test metrics are

$$R^2 = 1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}, \quad \text{MAE} = \frac{1}{N} \sum_i |y_i - \hat{y}_i|,$$

together with Pearson’s r between y and $\hat{y}$. Higher R^2 and Pearson r are better; lower MAE is better.

Four worked examples follow, one per probe type and per record composition. Examples 1 and 3 come from the purely numerical benchmark; Examples 2 and 4 come from the mixed-type datasets of

App. E.3, where most of a record is words rather than digits. In each case the upper panel is exactly the string handed to the frozen decoder and the lower panel states what the probe returns and what it is scored against.

Example 1: MiniBooNE — classification probe (purely numerical record)

Direct LM input (record abridged; label absent)

<ParticleID_0: 4.192> ...<ParticleID_49: 0.2366>

Probe and scoring

Representation passed to scikit-learn: standardized vector $\tilde{z}$.

Probe output: probability $p_\phi(y = 1 | x)$; the evaluator predicts $\hat{y} = \mathbf{1}[p_\phi(y = 1 | x) \geq 0.5]$.

Gold target, used only for scoring: $y = 1$.

Example 2: Census-KDD — classification probe (95.3% of value characters non-numeric)

Direct LM input (record abridged; label V42 absent)

<V1: 65> <V2: Not in universe> <V5: 12th grade no diploma>
<V8: Married-civilian spouse present> <V11: Black> <V13:
Female> ...
<V35: United-States> <V36: Native- Born in the United States>
<V41: 94>

Probe and scoring

Representation passed to scikit-learn: standardized vector $\tilde{z}$.

Probe output: probability $p_\phi(y = 1 | x)$; the evaluator predicts $\hat{y} = \mathbf{1}[p_\phi(y = 1 | x) \geq 0.5]$.

Gold target, used only for scoring: $y = 0$, the - 50000 . income class.

Example 3: Superconductivity — regression probe (purely numerical record)

Direct LM input (record abridged; target criticaltemp absent)

<mean_atomic_mass: 88.94> ...<wtd_std_Valence: 0.4432>

Probe and scoring

Representation passed to scikit-learn: standardized vector $\tilde{z}$.

Probe output: checkpoint-dependent scalar $\hat{y}_\psi(x)$.

Gold target, used only for scoring: $y = 9.5$.

Example 4: NYC-Taxi — regression probe (3.0% of value characters non-numeric)

Direct LM input (record abridged; target tip_amount absent)

```
<VendorID: 2> <store_and_fwd_flag: N> <RatecodeID: 1>  
<PULocationID: 97>  
<DOLocationID: 61> <passenger_count: 1> <extra: 0> <mta_tax:  
0.5> ...  
<total_amount: 11.8> <lpep_dropoff_datetime_minute: 5>
```

Probe and scoring

Representation passed to scikit-learn: standardized vector $\tilde{z}$.
Probe output: checkpoint-dependent scalar $\hat{y}_\psi(x)$.
Gold target, used only for scoring: $y = 1.0$.

All checkpoints use identical record inputs and data splits. EMB-8 is the unweighted mean of the two classification ROC-AUC values and six regression R^2 values; because these metrics have different semantics, the aggregate is a compact summary rather than a substitute for per-dataset results.

Embedding diagnostics include effective rank, mean pairwise cosine, metric faithfulness, and CKA. Effective rank is $\exp(-\sum_j p_j \log p_j)$ for the normalized eigenvalues of the centered embedding covariance; larger values indicate that variation occupies more directions. Mean pairwise cosine measures anisotropy; values near one mean records point in nearly the same direction. Metric faithfulness is the Spearman correlation between cosine distance and absolute target difference over sampled test pairs. Linear CKA measures similarity between the spaces produced by two checkpoints.

Generation axis. The generation axis evaluates the frozen decoder directly, with no fitted probe. On each dataset we use 500 held-out records. Masked-value imputation and field lookup use the original 320-token decoder harness, greedy decoding, at most 12 new tokens, and deterministic numeric parsing; parse failures count as incorrect. Held-out likelihood uses the same 320-token input cap. Target emission is a separate, training-matched diagnostic with a 496-token cap and is valid only for the four datasets whose complete prompt fits that budget. The four tasks isolate target prediction, missing-value inference, copying, and distribution modeling, respectively.

- **Masked-value imputation.** We deterministically choose one field, truncate the serialization immediately after its key marker, and ask the model to continue with the missing value. The primary metric is within-10% accuracy:

$$\frac{1}{N} \sum_{i=1}^N \mathbf{1} [|\hat{v}_i - v_i| \leq 0.1 \max(|v_i|, 10^{-6})].$$

Exact match after four-significant-figure normalization, MAE, median absolute error, and parse-failure rate are secondary metrics. The median floor predicts each field’s train-split median conditional on field identity.

Example 5 shows one such prompt.

Example 5: MiniBooNE — masked-value imputation (no fitted probe)

Direct LM prompt (record prefix abridged; masked value absent)

```
<ParticleID_0: 4.192> ...<ParticleID_35:
```

Completion and scoring

LM completion: checkpoint-dependent continuation text.
Parsed prediction: first number in the continuation, denoted $\hat{v}$.
Gold feature value, used only for scoring: $v = 0.3543$.

To preserve comparability with the main results, GEN-8 is the unweighted mean of the within-10% imputation accuracy over the eight datasets. Target-generation results are reported separately because classification accuracy and regression log-ratio error are not commensurate.

Held-out record perplexity remains under the generation axis because it evaluates the record distribution rather than an external capability.

Table 8: Dataset-to-metric map. Every dataset uses masked-value imputation, field lookup, and perplexity; target emission is reported only where the complete training-matched prompt fits the 496-token budget. Embedding metrics depend on whether the target is categorical or continuous.

Dataset	Prediction target	Task type	Embedding metrics	Generation metrics
MiniBooNE	signal	binary classification	Acc., ROC-AUC $\uparrow$	impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
HIGGS	class	binary classification	Acc., ROC-AUC $\uparrow$	target Acc. $\uparrow$; impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
Superconductivity	critical temperature	regression	R^2 , r $\uparrow$; MAE $\downarrow$	impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
USDA FoodData	energy (kcal)	regression	R^2 , r $\uparrow$; MAE $\downarrow$	impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
CPU activity	user CPU %	regression	R^2 , r $\uparrow$; MAE $\downarrow$	target log error $\downarrow$; impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
Naval propulsion	compressor decay	regression	R^2 , r $\uparrow$; MAE $\downarrow$	target log error $\downarrow$; impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
Protein structure	RMSD	regression	R^2 , r $\uparrow$; MAE $\downarrow$	target log error $\downarrow$; impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$
YearPredictionMSD	year	regression	R^2 , r $\uparrow$; MAE $\downarrow$	impute $\uparrow$; lookup $\uparrow$; PPL $\downarrow$

D EXPERIMENT DETAILS

D.1 MODEL ARCHITECTURES

Tab. 9 summarizes the checkpoints used in the primary experiments and model-size robustness sweep. The Qwen3.5 and Qwen3.8 checkpoints use the same hybrid decoder family; the vision components exposed by these checkpoints are unused, and all experiments operate on the text decoder only.

Table 9: Model checkpoints used in our experiments. Vocabulary denotes tokenizer vocabulary size.

Checkpoint	Scale	Decoder architecture	Vocabulary
Qwen2.5-1.5B-Instruct	1.5B	Dense Qwen2 decoder	151,936
Qwen3-4B-Instruct-2507	4B	Dense Qwen3 decoder	151,936
Qwen3.5-9B	9.6B	32-layer hybrid full/linear-attention decoder	248,320
Qwen3.8-27B	27B	64-layer hybrid full/linear-attention decoder	248,320

D.2 COMPLETE ARM REGISTRY

Table 10: The principal arms. “Views” is the number of deterministic row serializations per epoch. All MODEL recipes use weight 0.5 for that loss. Every serialization is right-truncated at 512 tokens; a “permuted-prefix view” denotes independent field-order permutation followed by truncation. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Recipe	Stage-1 order / views	Stage-1 objective	Stage 2	Budget	Purpose
Frozen decoder	fixed-order / 1	none	–	–	frozen initialization
MODEL	fixed-order / 1	MODEL	–	2 epochs	primary baseline
MATCH only	fixed-order / 1	MATCH only (disjoint views)	–	2 epochs	consistency-only control
MODEL + MATCH	fixed-order / 1	MODEL + MATCH (disjoint views)	–	2 epochs	MODEL+MATCH recipe
TUNE only	fixed-order / 1	none	fixed-order TUNE	TUNE only	no-Stage-1 control
Extended-compute MODEL	fixed-order / 1	MODEL	–	20 epochs	10× compute
Field-permuted MODEL	field-permuted / 10	MODEL	–	2 epochs	field-permuted baseline
Field-permuted MATCH only	field-permuted / 10	MATCH only (permuted-prefix views)	–	2 epochs	consistency-only control
Field-permuted MODEL + MATCH	field-permuted / 10	MODEL + MATCH (permuted-prefix views)	–	2 epochs	field-permuted MODEL+MATCH recipe
MODEL → TUNE	fixed-order / 1	MODEL	fixed-order TUNE	2 + 2 epochs	supervised baseline
MODEL + MATCH → TUNE	fixed-order / 1	MODEL + MATCH (disjoint views)	fixed-order TUNE	2 + 2 epochs	supervised MODEL+MATCH recipe
Field-permuted MODEL → TUNE	field-permuted / 10	MODEL	fixed-order TUNE	2 + 2 epochs	supervised control
Field-permuted MODEL + MATCH → TUNE	field-permuted / 10	MODEL + MATCH (permuted-prefix views)	fixed-order TUNE	2 + 2 epochs	robustness-oriented instantiation
Field-permuted MODEL → field-permuted TUNE	field-permuted / 10	MODEL	field-permuted TUNE	matched steps	order-matched control
Field-permuted MODEL + MATCH → field-permuted TUNE	field-permuted / 10	MODEL + MATCH (permuted-prefix views)	field-permuted TUNE	matched steps	order-matched recipe
Single-view field-permuted MODEL	field-permuted / 1	MODEL	–	standard MODEL steps	pure order effect
Matched-update field-permuted MODEL	field-permuted / 10	MODEL	–	standard MODEL steps	field-order permutation at fixed compute
Legacy-serialization MODEL + MATCH	legacy / 1	MODEL + MATCH (disjoint views)	–	2 epochs	original serialization
MODEL + order-aware rank	fixed-order / 1	MODEL + order-aware rank	–	2 epochs	order-aware alternative

The complete sweep also contains TUNE descendants of the consistency-only and order-aware arms. At 1.5B, MATCH is directionally positive in the four model-and-match comparisons, but only the MODEL+MATCH→TUNE comparison against MODEL→TUNE exceeds the embedding duplicate band; the aligned larger-scale aggregate effects remain small. Comparing the two field-permuted Stage-1 parents under standard versus field-permuted TUNE isolates the supervised presentation; at 1.5B, the standard presentation improves EMB-8 by 0.012/0.014 (Tab. 12).

D.3 TRAINING DETAILS

Every standard Stage-1 arm uses two data epochs, effective batch 16, AdamW ($\beta_1 = 0.9, \beta_2 = 0.95$, weight decay 0.1), learning rate 10^{-5} , 100 warmup steps, cosine decay, maximum length 512, bfloat16, and seed 42. Larger models use FSDP while preserving the same effective batch. Appx. D.2 contains the complete arm registry.

The 512-token budget truncates some serialized records. On MiniBooNE, all 3,000 test rows contain 590–726 tokens (mean 708.5), so every row is truncated; the encoded prefix contains 35.8 of 50 complete fields on average, and 91.5% of raw truncations end inside the next key-value unit.

We also audited the field-order permutation construction on the first 3,000 MiniBooNE training rows, using ten deterministic pairs per row (30,000 pairs total). Every pair exposed different complete-field sets; each view exposed 35.7 of 50 complete fields on average, the mean set Jaccard similarity was 0.556, and the mean symmetric difference between paired prefixes was 20.4 fields. These measurements confirm that permuted-prefix views change both field order and visible content.

E EXTENDED OVERALL RESULTS

This section provides the complete controls and per-dataset metrics underlying the compact main-paper comparisons.

E.1 MATCHED RECIPE AND ABLATION CONTROLS

Table 11: **Matched complete-record comparison for the final MATCH intervention.** Both recipes use field-permuted MODEL followed by the same TUNE interface; the second additionally uses MATCH. The near-identical mean motivates evaluating Match through its partial-view robustness role rather than complete-record aggregate accuracy. Higher is better.

Recipe	MiniBooNE	HIGGS	Supercond.	YearPred	Naval	CPU act.	Protein	USDA	Mean 8
Field-permuted MODEL → TUNE	.963	.697	.817	.224	.960	.970	.424	.969	.753
Field-permuted MODEL + MATCH → TUNE	.968	.691	.806	.219	.972	.965	.434	.967	.753

Table 12: **Aggregate controls used in the ablation discussion.** EMB-8 is the eight-dataset frozen-probe aggregate; imputation is the corresponding eight-dataset within-10% accuracy. The final two rows use field-permuted records during TUNE to isolate the supervised interface. Higher is better.

Recipe	EMB-8 $\uparrow$	Imputation $\uparrow$
Frozen decoder	.688	.106
MODEL	.730	.401
MATCH only	.614	.033
MODEL+MATCH	.733	.401
Field-permuted MODEL	.739	.448
Field-permuted MODEL+MATCH	.740	.445
MODEL $\rightarrow$ TUNE	.736	.145
MODEL+MATCH $\rightarrow$ TUNE	.743	.145
Field-permuted MODEL $\rightarrow$ TUNE	.753	.221
Field-permuted MODEL+MATCH $\rightarrow$ TUNE	.753	.234
Field-permuted MODEL $\rightarrow$ field-permuted TUNE	.741	–
Field-permuted MODEL+MATCH $\rightarrow$ field-permuted TUNE	.740	–

E.1.1 SENSITIVITY TO THE LOSS WEIGHTS

Eq. equation 5 fixes one pair of coefficients. Writing the MODEL+MATCH objective in general form as

$$\mathcal{L}_{\text{Model+Match}} = \lambda_1 \mathcal{L}_{\text{record}} + \lambda_2 \mathcal{L}_{\text{cons}}, \quad (7)$$

the reported recipe is $\lambda_1 = 0.5$, $\lambda_2 = 1.0$. Because Adam normalizes each parameter’s update by its own gradient scale, multiplying both coefficients by a constant leaves the update nearly unchanged, so the operative hyperparameter is the ratio λ_2/λ_1 . We therefore sweep that ratio at fixed $\lambda_1 = 0.5$ and include one row at doubled scale to test the reduction.

Table 13: **The reported consistency weight is at the optimum on the dataset where MATCH has an effect, and irrelevant on one where it does not.** Primary embedding metric (R^2) after MODEL+MATCH at 1.5B with permuted-prefix views, varying only the loss coefficients of Eq. equation 7. The reported setting is in bold. The last two rows are controls: $\lambda_1=0$ removes the record term, and $\lambda_1=1.0$, $\lambda_2=2.0$ preserves the ratio at doubled scale. The same-configuration measurement spread on this metric is 0.005, so differences below that are not resolved.

λ_1	λ_2	Protein $R^2 \uparrow$	USDA $R^2 \uparrow$
0.5	0.00	.396	.942
0.5	0.25	.411	.940
0.5	0.50	.415	.939
0.5	1.00	.420	.937
0.5	2.00	.416	.938
0.5	4.00	.414	.938
0.0	1.00	.248	.878
1.0	2.00	.420	.938

On Protein, where MATCH changes the embedding metric, the sweep is single-peaked at the reported coefficients: R^2 rises from .396 at $\lambda_2=0$ to .420 at $\lambda_2=1.0$, then falls to .414 at $\lambda_2=4.0$. Halving or quadrupling λ_2 costs .005 and .006 respectively, while removing the term costs .024; the choice is therefore neither arbitrary nor delicate, since any $\lambda_2 \in [0.5, 2.0]$ stays within .005 of the best value. On USDA every setting lies within measurement spread of every other, including the removal of MATCH altogether. Read together, the two columns indicate that λ_2 is worth tuning only where the consistency term is active at all, and they explain why MATCH looks inert in aggregate: averaging a dataset where it is worth +.024 with several where it is worth nothing yields a small mean. Held-out record perplexity is unchanged across the swept rows (2.13–2.14 on Protein), so the gain is not accompanied by worse held-out record likelihood. The two control rows behave as expected: dropping

$\mathcal{L}_{\text{record}}$ collapses the representation (R^2 .248, perplexity 92.6), confirming that no weighting of $\mathcal{L}_{\text{cons}}$ substitutes for the record term, and doubling both coefficients reproduces the reported row to within .001, supporting the reduction to a single ratio. This sweep covers two regression datasets at one scale and one seed, so it bounds the sensitivity of the reported setting rather than establishing a globally optimal weight.

E.2 COMPLETE PER-DATASET TARGET-PREDICTION RESULTS

Tabs. 14–17 report the complete per-dataset results. *Resources* gives parameter count for neural models, the input resource for non-neural baselines, and API access for Cohere. The frozen-decoder recipe and Qwen2.5-1.5B are the same checkpoint. The Cohere probe uses 10,000 training records; all other zero-shot probes use 24,000.

Table 14: Detailed classification-probe results. ACC and ROC–AUC are reported for MiniBooNE and HIGGS; higher is better. Mean is the arithmetic mean of the four displayed metrics. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model / recipe	Resources	MiniBooNE		HIGGS		Mean $\uparrow$
		ACC $\uparrow$	ROC–AUC $\uparrow$	ACC $\uparrow$	ROC–AUC $\uparrow$	
<i>floors / ceilings</i>						
F2: char-TF-IDF probe floor	50k sparse features	0.833	0.890	0.554	0.552	.707
C1: XGBoost raw-feature ceiling	raw features	0.940	0.983	0.706	0.788	.854
<i>zero-shot decoder LLMs (18 models)</i>						
Qwen2.5-1.5B	1.5B	0.898	0.956	0.617	0.655	.781
Qwen3-1.7B	1.7B	0.896	0.957	0.609	0.651	.778
Qwen3-4B	4B	0.904	0.960	0.607	0.642	.778
Qwen2.5-7B	7B	0.900	0.960	0.604	0.644	.777
Qwen3-8B	8B	0.897	0.955	0.609	0.636	.774
Qwen3-14B	14B	0.901	0.959	0.621	0.660	.785
Mistral-7B-v0.3	7B	0.905	0.958	0.612	0.647	.780
OLMo-2-7B	7B	0.898	0.961	0.667	0.725	.813
phi-4 (14B)	14B	0.901	0.958	0.662	0.722	.811
DeepSeek-V2-Lite (16B MoE)	16B MoE	0.892	0.958	0.606	0.646	.775
gpt-oss-20b	20B MoE	0.891	0.954	0.618	0.668	.783
SmolLM3-3B	3B	0.901	0.959	0.646	0.705	.803
Falcon3-7B-Base	7B	0.900	0.959	0.566	0.607	.758
granite-3.3-8b-base	8B	0.901	0.961	0.615	0.655	.783
Mistral-Nemo-Base-12B	12B	0.896	0.956	0.610	0.662	.781
gemma-4-12B	12B	0.896	0.957	0.601	0.639	.773
Llama-3.1-8B	8B	0.900	0.960	0.657	0.712	.807
Qwen3.5-9B	9.6B	0.892	0.954	0.601	0.644	.773
<i>purpose-built embedders (10 models)</i>						
bge-small-en-v1.5	33M	0.838	0.905	0.560	0.580	.721
all-MiniLM-L6-v2	22M	0.812	0.870	0.577	0.596	.714
gte-Qwen2-1.5B-instruct	1.5B	0.857	0.913	0.558	0.569	.724
e5-mistral-7b-instruct	7B	0.850	0.906	0.556	0.571	.721
Qwen3-Embedding-0.6B	0.6B	0.881	0.943	0.575	0.598	.749
Qwen3-Embedding-4B	4B	0.884	0.943	0.593	0.620	.760
Qwen3-Embedding-8B	8B	0.885	0.944	0.585	0.620	.758
gte-Qwen2-7B-instruct	7B	0.865	0.923	0.564	0.588	.735
NV-Embed-v2	7B	0.891	0.952	0.576	0.608	.757
LLM2Vec-Mistral-7B	7B	0.892	0.955	0.588	0.619	.763
<i>commercial embedding API (selected)</i>						
cohere.embed-v4 (Bedrock)	API	0.868	0.930	0.616	0.664	.770
<i>trained 1.5B recipes</i>						
Frozen decoder	1.5B	0.898	0.956	0.617	0.655	.781
MODEL	1.5B	0.911	0.968	0.628	0.675	.795
TUNE only	1.5B	0.907	0.963	0.629	0.682	.795
MODEL $\rightarrow$ TUNE	1.5B	0.917	0.969	0.633	0.691	.802
MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.913	0.969	0.636	0.695	.803
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.913	0.968	0.630	0.691	.801

Table 15: Detailed regression-probe R^2 by dataset; higher is better. Mean is the arithmetic mean over the six displayed datasets. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model / recipe	Resources	Supercond.	USDA	CPU	Naval	Protein	YearPred	Mean $\uparrow$
<i>floors / ceilings</i>								
F2: char-TF-IDF probe floor	50k sparse features	0.865	0.860	0.281	0.904	0.201	0.005	.519
C1: XGBoost raw-feature ceiling	raw features	0.916	0.997	0.985	0.996	0.623	0.277	.799
<i>zero-shot decoder LLMs (18 models)</i>								
Qwen2.5-1.5B	1.5B	0.769	0.910	0.891	0.860	0.311	0.152	.649
Qwen3-1.7B	1.7B	0.778	0.920	0.886	0.885	0.316	0.155	.657
Qwen3-4B	4B	0.783	0.940	0.926	0.893	0.332	0.160	.672
Qwen2.5-7B	7B	0.814	0.939	0.945	0.927	0.323	0.158	.684
Qwen3-8B	8B	0.797	0.946	0.931	0.917	0.330	0.172	.682
Qwen3-14B	14B	0.809	0.951	0.947	0.929	0.342	0.156	.689
Mistral-7B-v0.3	7B	0.813	0.950	0.948	0.929	0.355	0.172	.695
OLMo-2-7B	7B	0.817	0.941	0.921	0.930	0.329	0.160	.683
phi-4 (14B)	14B	0.810	0.954	0.923	0.933	0.340	0.143	.684
DeepSeek-V2-Lite (16B MoE)	16B MoE	0.778	0.926	0.921	0.837	0.334	0.167	.661
gpt-oss-20b	20B MoE	0.753	0.915	0.803	0.830	0.284	0.153	.623
SmolLM3-3B	3B	0.780	0.913	0.876	0.876	0.277	0.159	.647
Falcon3-7B-Base	7B	0.799	0.930	0.921	0.924	0.350	0.158	.680
granite-3.3-8b-base	8B	0.805	0.935	0.945	0.901	0.351	0.155	.682
Mistral-Nemo-Base-12B	12B	0.810	0.945	0.944	0.944	0.348	0.152	.691
gemma-4-12B	12B	0.813	0.952	0.922	0.921	0.353	0.144	.684
Llama-3.1-8B	8B	0.804	0.946	0.911	0.915	0.321	0.174	.679
Qwen3.5-9B	9.6B	0.807	0.939	0.928	0.897	0.349	0.149	.678
<i>purpose-built embedders (10 models)</i>								
bge-small-en-v1.5	33M	0.562	0.673	0.611	0.608	0.088	0.102	.441
all-MiniLM-L6-v2	22M	0.623	0.664	0.683	0.624	0.110	0.026	.455
gte-Qwen2-1.5B-instruct	1.5B	0.574	0.670	0.954	0.745	0.258	0.014	.536
e5-mistral-7b-instruct	7B	0.662	0.662	0.942	0.810	0.310	0.013	.567
Qwen3-Embedding-0.6B	0.6B	0.668	0.781	0.755	0.743	0.195	0.117	.543
Qwen3-Embedding-4B	4B	0.690	0.868	0.868	0.716	0.257	0.110	.585
Qwen3-Embedding-8B	8B	0.770	0.903	0.893	0.849	0.275	0.118	.635
gte-Qwen2-7B-instruct	7B	0.642	0.692	0.945	0.795	0.325	0.017	.569
NV-Embed-v2	7B	0.803	0.916	0.932	0.922	0.323	0.139	.673
LLM2Vec-Mistral-7B	7B	0.812	0.904	0.902	0.944	0.318	0.162	.674
<i>commercial embedding API (selected)</i>								
cohere.embed-v4 (Bedrock)	API	0.770	0.863	0.764	0.871	0.200	0.126	.599
<i>trained 1.5B recipes</i>								
Frozen decoder	1.5B	0.769	0.910	0.891	0.860	0.311	0.152	.649
MODEL	1.5B	0.808	0.914	0.954	0.920	0.390	0.213	.700
TUNE only	1.5B	0.773	0.939	0.904	0.884	0.380	0.185	.678
MODEL $\rightarrow$ TUNE	1.5B	0.804	0.943	0.934	0.946	0.401	0.198	.704
MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.797	0.950	0.949	0.959	0.413	0.211	.713
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.806	0.967	0.965	0.972	0.434	0.219	.727

Table 16: Detailed regression-probe MAE by dataset; lower is better. Naval MAE is multiplied by 10^4 ; other columns retain their target units. Mean is the arithmetic mean of these displayed values and is a descriptive aggregate because units differ across datasets. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model / recipe	Resources	Supercond.	USDA	CPU	Naval $\times 10^4$	Protein	YearPred	Mean $\downarrow$
<i>floors / ceilings</i>								
F2: char-TF-IDF probe floor	50k sparse features	8.60	44.30	8.83	26.03	4.46	8.13	16.725
C1: XGBoost raw-feature ceiling	raw features	5.59	4.75	1.48	6.57	2.70	6.62	4.618
<i>zero-shot decoder LLMs (18 models)</i>								
Qwen2.5-1.5B	1.5B	12.10	35.93	4.29	41.12	4.11	7.33	17.480
Qwen3-1.7B	1.7B	11.84	34.51	4.45	37.67	4.10	7.32	16.648
Qwen3-4B	4B	11.63	29.29	3.60	35.96	4.08	7.27	15.305
Qwen2.5-7B	7B	10.58	28.97	3.09	28.30	4.10	7.30	13.723
Qwen3-8B	8B	11.17	27.06	3.50	31.27	4.08	7.30	14.063
Qwen3-14B	14B	10.56	25.84	3.06	28.53	4.02	7.36	13.228
Mistral-7B-v0.3	7B	10.61	26.14	3.00	28.40	3.98	7.26	13.232
OLMo-2-7B	7B	10.36	28.95	3.75	28.93	4.05	7.28	13.887
phi-4 (14B)	14B	10.57	25.87	3.62	27.26	4.04	7.42	13.130
DeepSeek-V2-Lite (16B MoE)	16B MoE	11.97	33.29	3.68	44.94	4.01	7.22	17.518
gpt-oss-20b	20B MoE	12.51	37.31	5.93	44.57	4.23	7.36	18.652
SmolLM3-3B	3B	11.76	35.98	4.63	38.28	4.23	7.21	17.015
Falcon3-7B-Base	7B	11.08	32.48	3.66	30.08	3.98	7.31	14.765
granite-3.3-8b-base	8B	10.90	29.12	3.02	34.57	3.99	7.33	14.822
Mistral-Nemo-Base-12B	12B	10.56	27.65	3.11	26.12	3.99	7.29	13.120
gemma-4-12B	12B	10.72	26.17	3.62	30.94	3.98	7.39	13.803
Llama-3.1-8B	8B	10.97	26.96	3.80	32.27	4.05	7.20	14.208
Qwen3.5-9B	9.6B	10.90	30.13	3.54	34.78	3.98	7.38	15.118
<i>purpose-built embedders (10 models)</i>								
bge-small-en-v1.5	33M	17.66	71.96	7.94	71.93	5.03	7.51	30.338
all-MiniLM-L6-v2	22M	16.24	72.18	7.22	69.68	4.96	8.00	29.713
gte-Qwen2-1.5B-instruct	1.5B	17.19	74.92	2.75	57.93	4.37	8.11	27.545
e5-mistral-7b-instruct	7B	14.56	73.33	3.11	48.02	4.15	8.09	25.210
Qwen3-Embedding-0.6B	0.6B	14.85	58.94	6.55	56.88	4.55	7.42	24.865
Qwen3-Embedding-4B	4B	14.20	43.52	4.89	58.65	4.32	7.46	22.173
Qwen3-Embedding-8B	8B	12.00	37.92	4.28	41.99	4.19	7.52	17.983
gte-Qwen2-7B-instruct	7B	15.04	70.13	3.06	50.89	4.10	8.00	25.203
NV-Embed-v2	7B	10.90	34.69	3.45	28.50	4.07	7.38	14.832
LLM2Vec-Mistral-7B	7B	10.75	36.29	4.11	25.30	4.08	7.27	14.633
<i>commercial embedding API (selected)</i>								
cohere.embed-v4 (Bedrock)	API	12.12	46.63	6.43	38.17	4.55	7.41	19.218
<i>trained 1.5B recipes</i>								
Frozen decoder	1.5B	12.10	35.93	4.29	41.12	4.11	7.33	17.480
MODEL	1.5B	10.72	35.60	2.80	30.03	3.81	6.99	14.992
TUNE only	1.5B	11.91	29.31	4.14	35.04	3.81	7.09	15.217
MODEL $\rightarrow$ TUNE	1.5B	10.94	29.08	3.42	25.01	3.76	7.00	13.202
MODEL + MATCH $\rightarrow$ TUNE	1.5B	11.11	26.51	2.94	20.86	3.68	6.93	12.005
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	10.51	20.42	2.43	18.05	3.64	6.96	10.335

Table 17: Detailed regression-probe Pearson r by dataset; higher is better. Mean is the arithmetic mean over the six displayed datasets. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model / recipe	Resources	Supercond.	USDA	CPU	Naval	Protein	YearPred	Mean $\uparrow$
<i>floors / ceilings</i>								
F2: char-TF-IDF probe floor	50k sparse features	0.930	0.928	0.538	0.951	0.459	0.081	.648
C1: XGBoost raw-feature ceiling	raw features	0.957	0.999	0.993	0.998	0.790	0.527	.877
<i>zero-shot decoder LLMs (18 models)</i>								
Qwen2.5-1.5B	1.5B	0.878	0.954	0.945	0.928	0.558	0.391	.776
Qwen3-1.7B	1.7B	0.883	0.959	0.942	0.941	0.564	0.395	.781
Qwen3-4B	4B	0.885	0.970	0.962	0.945	0.577	0.403	.790
Qwen2.5-7B	7B	0.903	0.969	0.972	0.963	0.569	0.401	.796
Qwen3-8B	8B	0.893	0.972	0.965	0.958	0.575	0.419	.797
Qwen3-14B	14B	0.901	0.975	0.973	0.964	0.586	0.403	.800
Mistral-7B-v0.3	7B	0.902	0.975	0.973	0.964	0.596	0.418	.805
OLMo-2-7B	7B	0.905	0.970	0.960	0.965	0.574	0.407	.797
phi-4 (14B)	14B	0.900	0.977	0.961	0.966	0.584	0.393	.797
DeepSeek-V2-Lite (16B MoE)	16B MoE	0.883	0.962	0.960	0.915	0.579	0.410	.785
gpt-oss-20b	20B MoE	0.868	0.957	0.897	0.911	0.534	0.399	.761
SmolLM3-3B	3B	0.884	0.956	0.937	0.936	0.528	0.402	.774
Falcon3-7B-Base	7B	0.895	0.965	0.960	0.961	0.593	0.401	.796
granite-3.3-8b-base	8B	0.898	0.967	0.972	0.950	0.593	0.399	.797
Mistral-Nemo-Base-12B	12B	0.901	0.972	0.972	0.972	0.591	0.395	.801
gemma-4-12B	12B	0.902	0.976	0.960	0.960	0.595	0.386	.797
Llama-3.1-8B	8B	0.897	0.973	0.955	0.957	0.569	0.419	.795
Qwen3.5-9B	9.6B	0.899	0.969	0.963	0.947	0.592	0.394	.794
<i>purpose-built embedders (10 models)</i>								
bge-small-en-v1.5	33M	0.750	0.821	0.785	0.780	0.300	0.322	.626
all-MiniLM-L6-v2	22M	0.790	0.816	0.829	0.791	0.333	0.163	.620
gte-Qwen2-1.5B-instruct	1.5B	0.758	0.819	0.977	0.863	0.509	0.144	.678
e5-mistral-7b-instruct	7B	0.814	0.816	0.971	0.900	0.557	0.163	.704
Qwen3-Embedding-0.6B	0.6B	0.819	0.884	0.869	0.863	0.444	0.342	.704
Qwen3-Embedding-4B	4B	0.833	0.932	0.933	0.848	0.510	0.338	.732
Qwen3-Embedding-8B	8B	0.878	0.950	0.945	0.922	0.532	0.350	.763
gte-Qwen2-7B-instruct	7B	0.802	0.832	0.972	0.891	0.570	0.187	.709
NV-Embed-v2	7B	0.897	0.957	0.965	0.960	0.568	0.383	.788
LLM2Vec-Mistral-7B	7B	0.901	0.951	0.950	0.972	0.564	0.408	.791
<i>commercial embedding API (selected)</i>								
cohere.embed-v4 (Bedrock)	API	0.878	0.929	0.875	0.933	0.448	0.360	.737
<i>trained 1.5B recipes</i>								
Frozen decoder	1.5B	0.878	0.954	0.945	0.928	0.558	0.391	.776
MODEL	1.5B	0.900	0.956	0.977	0.959	0.626	0.462	.813
TUNE only	1.5B	0.880	0.969	0.951	0.940	0.617	0.431	.798
MODEL $\rightarrow$ TUNE	1.5B	0.897	0.971	0.967	0.973	0.633	0.445	.814
MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.894	0.975	0.974	0.979	0.643	0.460	.821
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	0.899	0.983	0.983	0.986	0.659	0.468	.830

E.3 GENERALITY TO RECORDS WITH SUBSTANTIAL TEXT AND CATEGORICAL FIELDS

The eight datasets in Tab. 1 are numerical by construction: measured over field values, 272/272 fields are numeric, so the benchmark says nothing about records whose fields are largely words, which is common in real tabular data. We therefore re-ran the complete recipe block of Tab. 1 on two additional datasets, holding the recipe, the serialization, the batch size, and the two-epoch budget exactly as in the main experiments and changing only the data.

Census-KDD (OpenML 4535; 30,000 records, 41 fields, 28 of them categorical) is the text-heavy case: **95.3%** of the characters in a serialized record are non-numeric, with values such as `State government` and `Bachelors degree (BA AB BS)`. It is the direct test of whether the method survives text. **NYC-Taxi** (OpenML 42729, tip amount; 30,000 records, 18 fields, **9 of them declared categorical**) contains also text value: **3.0%** of its value characters are non-numeric, and its frozen zero-shot probe reaches $R^2 = 0.433$, leaving room for a training signal to register. Following Tab. 1, the classification dataset reports ROC-AUC and the regression dataset reports R^2 , and EMB-2 is their unweighted mean.

The method transfers to text-heavy records; resolving its components needs headroom. On Census-KDD, where more than nine in ten characters of a record are words, no recipe degrades the representation: every two-stage recipe lands within 0.006 ROC-AUC of the frozen 0.941 baseline,

Table 18: The complete MOMATUNE recipe block re-run on records that are not numerical, in the format of Tab. 1. The percentage after each dataset is the share of non-numeric characters in a serialized record, measured over field values. EMB-2 is the unweighted mean of the two columns. All trained recipes use a Qwen2.5-1.5B backbone, one seed, and the identical configuration reported in the main results; bold marks the best value in each column. On the text-heavy dataset every recipe stays at or above the frozen baseline and the best recipe exceeds it, but the recipe ordering is compressed because a frozen decoder already reaches 0.941 there. On the dataset with headroom the ordering of Tab. 1 reappears in full.

Model / recipe	Classification: ROC-AUC $\uparrow$	Regression: R^2 $\uparrow$	EMB-2 $\uparrow$
	Census-KDD (95.3% text)	NYC-Taxi (3.0% text)	
<i>Frozen decoder LMs (mean-pooled hidden states)</i>			
Qwen3-8B	.940	.444	.692
Gemma-4-12B	.937	.490	.713
<i>MOMATUNE framework (Qwen2.5-1.5B backbone)</i>			
Zero-shot frozen decoder	.941	.433	.687
TUNE only	.942	.437	.690
MODEL $\rightarrow$ TUNE	.940	.485	.712
MODEL + MATCH $\rightarrow$ TUNE	.936	.497	.717
Field-permuted MODEL $\rightarrow$ TUNE	.946	.522	.734
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	.945	.530	.738

and the best recipe, field-permuted MODEL $\rightarrow$ TUNE, reaches 0.946 – above the frozen 1.5B baseline and above both the frozen Qwen3-8B (0.940) and Gemma-4-12B (0.937) references. The gain is small in absolute terms (+0.005, just past the 0.005 same-configuration spread reported for Tab. 13), and the differences among recipes are not resolvable: a frozen decoder already scores 0.941 on this task, so little of the metric is available to move. That ceiling is a property of the task rather than of the field types, and it appears in the main results too – MiniBooNE moves from 0.956 to 0.969 while Protein, far from saturation, moves from 0.311 to 0.434.

NYC-Taxi isolates that confound. Its records are almost entirely numeric, but its frozen probe is far from the ceiling, and there the ordering of Tab. 1 reappears intact: MODEL $\rightarrow$ TUNE adds 0.052 R^2 over zero-shot while TUNE alone adds 0.004, field-order permutation adds a further 0.037, and MATCH adds 0.012 at fixed order and 0.008 under permutation – all above the 0.005 noise floor, with the field-permuted MATCH variant best at 0.530, the same row that is best in Tab. 1. Read together, the two datasets separate two questions that the original benchmark conflates: whether the method tolerates non-numeric fields, which these results support, and whether its individual components can be distinguished on a given task, which requires that the task not already be solved by a frozen decoder. We did not observe a case where text or categorical fields drove a recipe below the frozen baseline. The scope is two datasets, one backbone, and one seed, so these results bound the concern rather than settle the size of the effect on mixed-type data.

F COMPONENT EVIDENCE AND MECHANISM DIAGNOSTICS

This section groups the diagnostic evidence by the capability associated with each training component. The complete protocols follow the corresponding interpretation so that each claim, measurement, and boundary can be read together. The digit-sensitivity and incomplete-view analyses are scoped diagnostics; their dataset and sample scopes are stated with each result.

Paired record-bootstrap protocol. For the digit-sensitivity and field-dropout diagnostics, we use 10,000 paired percentile-bootstrap resamples generated with NumPy PCG64 seed 20260925. The record is the sampling unit, and each resample uses the same indices across compared recipes, perturbations, and views. ROC-AUC is recomputed within every resample while the fitted probe remains fixed. The resulting intervals quantify evaluation-record uncertainty conditional on the trained checkpoints, not variation across training seeds.

F.1 MODEL: NUMERICAL SENSITIVITY AND CROSS-FIELD CONTEXT

MODEL provides the broadest Stage-1 gains. It raises EMB-8 from 0.688 to 0.730 and masked-value completion from 0.106 to 0.401 (Tabs. 1 and 2). It also increases the correct-context gain from 0.058 to 0.167, while field-permuted MODEL reaches 0.271 (Tab. 3). These results show that the decoder becomes better at completing numerical values and relies more on the surrounding measurements from the same example.

The digit perturbation diagnostic asks whether representations respond more strongly to changes with larger numerical consequences. The leading-to-trailing sensitivity ratio rises from $2.08\times$ for the frozen checkpoint to $13.83\times$ after fixed-order MODEL and $17.82\times$ after field-permuted MODEL. The paired Model-minus-frozen difference is 11.75 (95% CI [7.04, 17.09]); the additional field-permutation difference is 3.99 ([-3.91, 13.17]) and is unresolved. This is a scoped sensitivity proxy rather than a claim of faithful magnitude geometry: it is evaluated on MiniBooNE, and the absolute cosine distances are small.

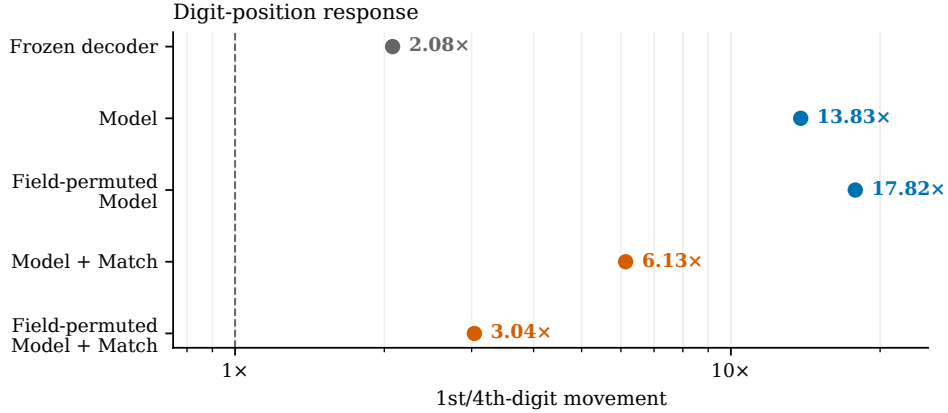


Figure 6: **Digit-position sensitivity across selected 1.5B arms.** MODEL produces the largest increase over the frozen checkpoint: both MODEL and field-permuted MODEL become more sensitive to a first-digit edit. Adding MATCH contracts the proxy for both MODEL and field-permuted MODEL. Every edited unit is complete in the encoded input and every variant changes one tokenizer token. MiniBooNE, 200 records; 95% paired record-bootstrap intervals are reported in Tab. 19. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

F.1.1 DIGIT-POSITION SENSITIVITY PROTOCOL AND FULL RESULTS

For each of 200 MiniBooNE test records, we select a numeric field only after tokenizing the fixed-order serialization. The selected key-value unit must be complete inside the 512-token input for the original and all four variants. We create four copies by changing one of its first four significant digits by one while keeping the serialized character length fixed; every variant must change the encoded token sequence, and at least one changed token must overlap the selected unit. All retained perturbations change exactly one token with the Qwen2.5 tokenizer. Let d_p be the cosine distance between the original embedding and the copy edited at position p . We report $\bar{d}_1/\bar{d}_4$, the ratio of mean distances across records, and the Spearman correlation between log absolute value change and embedding distance pooled over four dependent perturbations per record. The same records and perturbations are used for every checkpoint, with tokenizer identity checked against the reference token IDs. Because the absolute distances are small and the ratio can be sensitive to its denominator, we treat this as an exploratory diagnostic and do not interpret the distance scale. The raw case-level artifact, including token spans, IDs, and model hashes, is `learnings/digit_sensitivity_corrected_20260913.json`.

Table 19: Complete MiniBooNE digit-position diagnostic. The input first-versus-fourth value-change ratio is approximately $1000\times$; it is a reference contrast, not an expected embedding ratio. Larger ratios indicate greater leading-digit sensitivity under this proxy. Intervals use 10,000 paired percentile-bootstrap resamples over records and are conditional on the evaluated checkpoints.

Recipe	$\bar{d}_1/\bar{d}_4 \uparrow$	95% CI	Spearman $\rho \uparrow$
Frozen decoder	2.078	[1.135, 3.761]	0.261
MODEL	13.832	[9.363, 19.260]	0.342
MATCH only	1.008	[0.871, 1.162]	-0.011
MODEL + MATCH	6.134	[4.472, 8.106]	0.333
Field-permuted MODEL	17.822	[9.559, 28.420]	0.394
Field-permuted MODEL + MATCH	3.038	[1.266, 8.038]	0.376
MODEL $\rightarrow$ TUNE	4.434	[3.217, 5.803]	0.413
MODEL + MATCH $\rightarrow$ TUNE	3.698	[2.127, 5.960]	0.389
Underlying value change	1000.00	–	–

F.2 MATCH: STABILITY AND INCOMPLETE-RECORD PREDICTION

MATCH consistently contracts paired views, but whether that stability remains predictive depends on the view construction. With permuted-prefix training, adding MATCH reduces full-to-incomplete displacement from 0.00468 to 0.000306 and raises mean incomplete-view AUC from 0.7192 to 0.7584. The paired displacement difference is -0.004372 (95% CI $[-0.004390, -0.004355]$), equivalent to a $15.305\times$ reduction (95% CI $[15.257\times, 15.354\times]$); the paired incomplete-view AUC difference is $+0.0392$ ($[0.0233, 0.0555]$). The full-view AUC difference is unresolved at $+0.00145$ ($[-0.00453, 0.00763]$). With disjoint-half views, displacement also falls, but incomplete-view AUC drops from 0.5841 to 0.5331, a paired difference of -0.0510 ($[-0.0822, -0.0200]$). A plausible explanation is that permuted-prefix views retain overlapping conditional information, whereas disjoint halves can contain incompatible predictive fields. The current experiment supports this view-compatibility hypothesis but does not establish a general rule for constructing view pairs.

MATCH should therefore be interpreted as a conditional robustness regularizer, not a general complete-record accuracy stage. Accordingly, MATCH is not required by the core MOMATUNE path; it is an optional extension for settings in which robustness across partial record views is a deployment requirement. The matched field-permuted recipes reach the same EMB-8 after TUNE (Tab. 11), while Stage-1 completion changes only from 0.448 to 0.445 (Tab. 2).

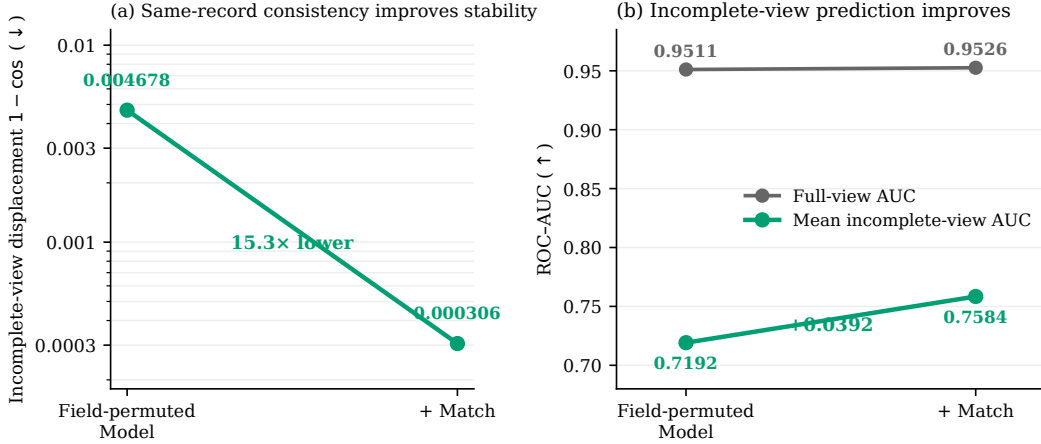


Figure 7: **Same-record consistency improves incomplete-record stability and prediction when trained with permuted-prefix views.** The matched field-permuted MODEL to field-permuted MODEL+MATCH comparison uses identical evaluation views and differs only by MATCH. Panel (a) reports incomplete-view displacement; lower is better. Panel (b) reports full-view and mean incomplete-view ROC–AUC from a logistic probe fitted on full-view embeddings; higher is better. MATCH reduces displacement by $15.305\times$ (95% CI $[15.257\times, 15.354\times]$) and raises incomplete-view AUC by 0.0392 $[0.0233, 0.0555]$; the full-view AUC difference is $+0.00145$ $[-0.00453, 0.00763]$. MiniBooNE, 3,000 records.

F.2.1 FIELD-DROPOUT PROTOCOL AND FULL RESULTS

For each of 3,000 MiniBooNE test records, we first tokenize the raw fixed-order serialization to 512 tokens. The evaluation full view is rebuilt from the maximal prefix of complete key-value units visible in that encoding; if truncation cuts the next unit, that partial unit is removed. We then partition only these complete units into two deterministic, complementary strict subsets while preserving their original order. Thus neither subset can introduce a field absent from its corresponding full view. The same seed-42 masks are shared by all arms. The *same-record margin* averages $\cos(z_A(i), z_B(i)) - \cos(z_A(i), z_B(j))$, where the different-record partner j comes from one fixed seed-7 permutation. Subtracting the cross-record term removes the dominant record-independent cosine component. *Dropout displacement* is the record mean of $1 - \cos(z_{\text{full}}, z_{\text{subset}})$ averaged over both complementary subsets. For task retention, logistic regression with fixed $C = 0.1$ is fit on the first 2,000 full-view embeddings and evaluated on the remaining 1,000 records, once with full embeddings and once with each subset. “Half AUC” is the arithmetic mean of the two subset AUCs; retention is its ratio to full-view AUC. No alternative masks, pairings, probe splits, or training seeds are averaged. Probe sensitivity at $C \in \{0.01, 0.1, 1\}$ is retained in the raw artifact. The complete protocol, case plan, model hashes, per-record observations, and predictions are stored in `learnings/field_dropout_corrected_20260913.json`; its mask-plan SHA-256 is `ee700478b20b55d845e61e6c3cfa98272dbc8f70432e4a84cf3c4fc32e5528d4`. For displacement, paired resamples use all 3,000 records. For AUC, they use the 1,000 held-out records, preserve labels and both views under the same sampled indices, and recompute ROC–AUC in each resample with the fitted probe fixed.

Table 20: Corrected MiniBooNE field-dropout diagnostic. Lower displacement and higher margins indicate representation stability; full- and mean half-view AUC prevent collapsed representations from appearing robust. Each half is a strict subset of complete units in its fixed full view. Identical masks and pairings are used across recipes; the diagnostic contains 3,000 MiniBooNE records. The displayed 95% intervals use the paired record-bootstrap protocol above.

Recipe	Same-record margin $\uparrow$	Displacement $\downarrow$	Displacement 95% CI	Full AUC $\uparrow$	Half AUC $\uparrow$	Half AUC 95% CI	Retention $\uparrow$
Frozen decoder	-0.000243	0.002419	[0.002405, 0.002432]	0.9357	0.6937	[0.6727, 0.7148]	0.7414
MODEL	-0.002811	0.091574	[0.090951, 0.092195]	0.9547	0.5841	[0.5583, 0.6094]	0.6118
MATCH only	0.000000	0.00000306	[0.00000301, 0.00000310]	0.8903	0.7187	[0.6999, 0.7372]	0.8072
MODEL + MATCH	0.000036	0.012001	[0.011934, 0.012068]	0.9515	0.5331	[0.5096, 0.5569]	0.5602
Field-permuted MODEL	-0.001106	0.004678	[0.004660, 0.004697]	0.9511	0.7192	[0.7034, 0.7352]	0.7562
Field-permuted MODEL + MATCH	-0.000027	0.000306	[0.000305, 0.000307]	0.9526	0.7584	[0.7420, 0.7754]	0.7962
MODEL $\rightarrow$ TUNE	-0.001142	0.013545	[0.013471, 0.013619]	0.9435	0.5475	[0.5217, 0.5729]	0.5803
MODEL + MATCH $\rightarrow$ TUNE	-0.000269	0.010406	[0.010365, 0.010447]	0.9491	0.5377	[0.5169, 0.5594]	0.5666

F.3 TUNE: TARGET SPECIALIZATION AND COMPLETION TRADE-OFF

TUNE uses labeled examples to expose the downstream target. It improves the primary target-prediction metric for every tested Stage-1 parent at 1.5B and remains beneficial for the field-permuted model-and-match parent through 27B (Fig. 4(c); Tab. 4). Returning to a fixed field order during TUNE also outperforms retaining field-permuted presentation: EMB-8 rises from 0.741 to 0.753 for the MODEL pipeline and from 0.740 to 0.753 for the model-and-match pipeline (Tab. 12).

This specialization does not improve every decoder capability. Across the four Stage-1 parents, masked-value completion falls from 0.401, 0.401, 0.448, and 0.445 to 0.145, 0.145, 0.221, and 0.234 after TUNE. The prompt-matched target-emission results in Appx. H confirm that the supervised checkpoints learn to emit their targets, but no recipe dominates every dataset. Applications requiring both target prediction and general record completion should therefore retain the Stage-1 checkpoint or use a retention-aware multi-task objective.

F.4 FIELD-ORDER PERMUTATION AND COMPUTE CONTROLS

Field-order permutation is a data intervention rather than an order-invariance test: after permutation, the fixed token budget exposes different field subsets and local contexts. The token-budget audit summarized below quantifies this effect, and the selected 1.5B ablations report the corresponding fixed-order, extended-compute, and field-permuted controls.

Appx. D.3 summarizes the MiniBooNE token-budget and paired-view audit. The case-level audit and tokenizer hashes are stored in `learnings/view-truncation-audit-20260913.json`. These measurements motivate the term *permuted-prefix views* and rule out an interpretation as complete, value-preserving field-order permutations.

Table 21: Selected 1.5B embedding ablations by dataset. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Arm	MB	HIGGS	Superc.	USDA	CPU	Naval	Protein	Year	Mean
Frozen	.956	.655	.769	.910	.891	.860	.311	.152	.688
MODEL	.968	.675	.808	.914	.954	.920	.390	.213	.730
MODEL + MATCH	.968	.671	.813	.915	.953	.923	.408	.210	.733
MATCH only	.941	.607	.670	.869	.873	.618	.242	.093	.614
MODEL, 10 $\times$ compute	.971	.691	.820	.920	.930	.936	.436	.245	.744
Field-permuted MODEL	.967	.664	.808	.942	.969	.948	.396	.215	.739
Field-permuted MODEL + MATCH	.969	.659	.807	.937	.969	.961	.420	.201	.740
MODEL + order-aware rank	.967	.685	.783	.932	.952	.901	.382	.195	.725

G ROBUSTNESS ACROSS MODEL SIZES

G.1 MODEL-SIZE ROBUSTNESS FROM 1.5B TO 27B

The main paper reports compact matched changes across four checkpoint scales. The figure below visualizes the target-prediction and completion trends, and the subsequent tables provide the complete values. Because checkpoint architecture, vocabulary, and tokenizer change with scale, these experiments establish robustness across the evaluated checkpoints rather than a fitted scaling law.

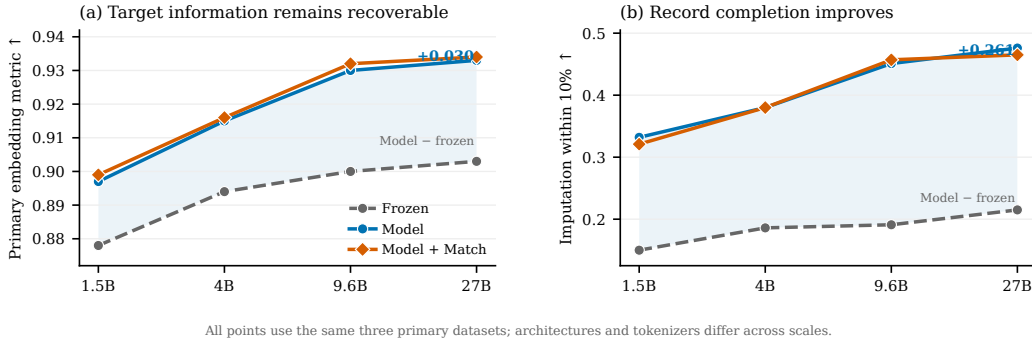


Figure 8: Fixed-order MODEL improves target prediction and record completion at every tested scale. Panel (a) reports the unweighted mean of MiniBooNE ROC–AUC, Superconductivity R^2 , and USDA R^2 ; panel (b) reports mean within-10% masked-value imputation on the same datasets. Gray dashed circles denote frozen checkpoints, blue circles denote fixed-order MODEL, and orange diamonds add MATCH. Relative to the frozen checkpoint, MODEL improves both metrics from 1.5B to 27B; adding MATCH produces only small and inconsistent changes. Lines connect separately trained checkpoint families whose architectures and tokenizers differ, so this is a one-seed robustness sweep rather than a fitted scaling law. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Tab. 22 preserves the full per-dataset version of the compact model-size summary in the main paper. The subsequent tables report the full eight-dataset ladder available through 9.6B. The 27B experiments were deliberately scoped to the aligned three primary datasets and are reported in Tabs. 22 and 26; the two dataset scopes are not mixed in a common mean.

Table 23: Full eight-dataset embedding results available through 9.6B: AUC for MiniBooNE/HIGGS, R^2 otherwise. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model	Post-training state	MB	HIGGS	Superc.	USDA	CPU	Naval	Protein	Year	Mean
1.5B	Frozen	.956	.655	.769	.910	.891	.860	.311	.152	.688
1.5B	MODEL	.968	.675	.808	.914	.954	.920	.390	.213	.730
4B	Frozen	.960	.642	.783	.940	.926	.893	.332	.160	.705
4B	MODEL	.970	.678	.830	.944	.961	.951	.433	.227	.749
9.6B	Frozen	.954	.644	.807	.939	.928	.897	.349	.149	.708
9.6B	MODEL	.974	.739	.857	.958	.970	.986	.506	.240	.779

Table 22: **Detailed model-size results for selected training recipes on the aligned three primary datasets.** MiniBooNE reports ROC–AUC, while Superconductivity and USDA report R^2 ; higher is better throughout, and means are unweighted over the same three datasets. All intended cells are complete. Because checkpoint architecture and vocabulary also change, this is a robustness sweep rather than a fitted scaling law. The compact main-paper counterpart is Tab. 4. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Training recipe	Scale	Embedding axis			
		MiniBooNE AUC $\uparrow$	Supercond. $R^2 \uparrow$	USDA $R^2 \uparrow$	Mean $\uparrow$
Frozen decoder	1.5B	.956	.769	.910	.878
	4B	.960	.783	.940	.894
	9.6B	.954	.807	.939	.900
	27B	.958	.804	.946	.903
MODEL	1.5B	.968	.808	.914	.897
	4B	.970	.830	.944	.915
	9.6B	.974	.857	.958	.930
	27B	.974	.865	.961	.933
TUNE only	1.5B	.963	.773	.939	.891
	4B	.967	.801	.964	.910
	9.6B	.970	.831	.978	.926
	27B	.966	.836	.978	.927
MODEL $\rightarrow$ TUNE	1.5B	.969	.804	.943	.905
	4B	.968	.821	.965	.918
	9.6B	.967	.858	.972	.932
	27B	.970	.849	.974	.931
Field-permuted MODEL + MATCH	1.5B	.969	.807	.937	.904
	4B	.970	.841	.954	.922
	9.6B	.971	.864	.966	.934
	27B	.970	.868	.970	.936
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	.968	.806	.967	.914
	4B	.971	.847	.973	.930
	9.6B	.973	.851	.980	.935
	27B	.973	.873	.979	.942

Table 24: Full eight-dataset masked-value-imputation results available through 9.6B. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Model	Post-training state	MB	HIGGS	Superc.	USDA	CPU	Naval	Protein	Year	Mean
Median floor	–	.242	.232	.276	.134	.242	.238	.282	.048	.212
1.5B	Frozen	.004	.126	.250	.196	.158	.086	.000	.028	.106
1.5B	MODEL	.214	.288	.494	.288	.366	.914	.576	.070	.401
4B	Frozen	.006	.130	.292	.260	.194	.144	.000	.024	.131
4B	MODEL	.226	.296	.572	.342	.388	.914	.618	.076	.429
9.6B	Frozen	.006	.172	.280	.286	.174	.122	.000	.034	.134
9.6B	MODEL	.316	.326	.662	.376	.394	.912	.646	.070	.463

G.2 MASKED-VALUE COMPLETION ACROSS MODEL SIZES

The eight-dataset table above gives the broad frozen-versus-MODEL comparison through 9.6B. The following aligned three-dataset table includes additional training recipes and the completed 27B checkpoints; the overlapping cells are retained to keep each scope self-contained. Masked-value imputation asks the decoder to regenerate a field value that has been blanked from the serialized record; a prediction counts as correct when it lands within 10% of the true value. These numbers

Table 25: **Masked-value imputation across model sizes for selected training recipes.** Values are within-10% accuracy on the aligned three primary datasets; higher is better, and the mean is unweighted. All intended cells are complete.

Training recipe	Scale	MiniBooNE $\uparrow$	Supercond. $\uparrow$	USDA $\uparrow$	Mean $\uparrow$
Frozen decoder	1.5B	.004	.250	.196	.150
	4B	.006	.292	.260	.186
	9.6B	.006	.280	.286	.191
	27B	.008	.350	.288	.215
MODEL	1.5B	.214	.494	.288	.332
	4B	.226	.572	.342	.380
	9.6B	.316	.662	.376	.451
	27B	.316	.680	.432	.476
TUNE only	1.5B	.016	.132	.090	.079
	4B	.014	.174	.108	.099
	9.6B	.010	.096	.048	.051
	27B	.014	.082	.100	.065
MODEL $\rightarrow$ TUNE	1.5B	.022	.168	.090	.093
	4B	.014	.168	.146	.109
	9.6B	.016	.120	.082	.073
	27B	.040	.072	.170	.094
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	1.5B	.034	.138	.060	.077
	4B	.032	.202	.022	.085
	9.6B	.034	.118	.042	.065
	27B	.218	.112	.020	.117

were previously carried alongside the embedding axis in Tab. 4; they are reported here instead to keep the main table focused on downstream prediction. Higher is better and the mean is unweighted over the same three primary datasets at every scale. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Only the frozen decoder and MODEL improve monotonically with scale here; every TUNE recipe remains well below MODEL alone, which is the expected cost of specializing the decoder to emit a single target rather than to model the whole record.

G.3 COMPLETE 27B RECIPE GRID

The complete grid separates the effects of MODEL, MATCH, TUNE, field-order permutation, and additional optimization on the default serialization at the largest evaluated scale.

Table 26: Complete 27B grid on the aligned three-primary-dataset scope. The embedding column is the mean primary downstream metric; imputation is within-10% accuracy. Higher is better.

Recipe	MODEL view	MATCH / TUNE	Embedding $\uparrow$	Imputation $\uparrow$
Frozen decoder	–	–	.903	.215
MODEL	default view	–	.933	.476
MODEL + MATCH	default view	MATCH	.934	.465
TUNE only	–	TUNE only	.927	.065
Extended-compute MODEL	default view, 10 $\times$	–	.920	.479
Field-permuted MODEL	permuted-prefix views	–	.936	.613
Field-permuted MODEL + MATCH	permuted-prefix views	MATCH	.936	.604
MODEL $\rightarrow$ TUNE	default view	TUNE	.931	.094
MODEL + MATCH $\rightarrow$ TUNE	default view	MATCH $\rightarrow$ TUNE	.928	.075
Field-permuted MODEL + MATCH $\rightarrow$ TUNE	permuted-prefix views	MATCH $\rightarrow$ TUNE	.942	.117

Across all four tested checkpoints, MODEL obtains higher embedding and masked-value-imputation aggregates than the corresponding frozen checkpoint. The complete recipe obtains higher target-prediction scores than TUNE alone at all four tested checkpoints and has the highest reported 27B embedding score (0.942). At equal update budgets, field-permuted MODEL also exceeds extended-compute MODEL at 27B by 0.016 on embedding and 0.134 on imputation. In contrast, adding MATCH to MODEL, field-permuted MODEL, or MODEL $\rightarrow$ TUNE remains aggregate-inert or negative through 27B, and MATCH does not improve imputation over its Stage-1 parent. Thus, MATCH contracts

paired views but is not a source of scale-growing aggregate accuracy; whether that contraction is useful remains view-dependent.

H ADDITIONAL DIRECT-DECODER EVALUATIONS

H.1 EVALUATION SCOPE AND PROMPT COMPATIBILITY

The currently reported held-out record perplexity, masked-value imputation, and field-lookup artifacts all use the original decoder harness’s code-level 320-token input cap. Target emission below is a separate diagnostic invocation: it uses the TUNE training template (`--gl_prompt train`) and a 496-token input cap (`--gl_max_input 496`). It is evaluated only on the four datasets for which the complete prompt is valid. The direct decoder aggregate in the main paper is eight-dataset masked-value imputation; target emission is reported separately on this prompt-valid subset.

H.2 PROMPT-MATCHED TARGET EMISSION

Target emission asks the decoder to produce the dataset target from the available fields. This is a secondary, prompt-conditioned diagnostic rather than the eight-dataset masked-value-imputation axis. Tab. 27 reports only the four datasets whose complete Stage-2 prompt, including the `Question: . . . Answer: suffix`, fits within the 496-token TUNE budget. HIGGS uses classification accuracy (higher is better); the three regression datasets use median absolute log ratio, median $|\log(\hat{y}/y)|$ (lower is better). Under the prompt used during TUNE, all four TUNE recipes beat the task-specific floor on all four datasets, but no recipe dominates: MODEL+MATCH→TUNE is best on HIGGS, CPU, and Protein, while the field-permuted complete recipe is best on Naval.

Table 27: Corrected 1.5B target-emission results on the four prompt-clean datasets. The prompt matches TUNE training and uses a 496-token budget. Bold marks the best TUNE recipe per dataset. “n/p” denotes no parseable numerical output, with the percentage of 500 evaluated records in parentheses. MODEL denotes next-token record adaptation; MATCH denotes optional same-record consistency; and TUNE denotes supervised fine-tuning (SFT).

Recipe	HIGGS Acc. ↑	CPU ↓	Naval ↓	Protein ↓
Task-specific floor	0.518	0.086	0.013	0.755
Frozen decoder	0.091	n/p (100%)	n/p (89%)	n/p (100%)
MODEL	0.014	n/p (100%)	n/p (100%)	n/p (100%)
TUNE only	0.720	0.013	0.002	0.310
MODEL → TUNE	0.714	0.022	0.002	0.301
MODEL + MATCH → TUNE	0.722	0.012	0.002	0.294
Field-permuted MODEL + MATCH → TUNE	0.708	0.015	0.001	0.341

MiniBooNE (718 tokens), USDA FoodData (597), YearPredictionMSD (977), and Superconductivity (1110) exceed the 496-token Stage-2 budget; right truncation deleted the question-and-answer suffix, so evaluating those checkpoints on that suffix would measure a train/test prompt mismatch. We exclude all four rather than treating their outputs as model failures. The remaining four datasets have maximum serializations of at most 362 tokens. The result is also prompt-sensitive. With the original 320-token instruction-style harness, TUNE only instead wins HIGGS, while the field-permuted complete recipe wins CPU, Naval, and Protein. All TUNE recipes still beat the corresponding floors on HIGGS, Naval, and Protein, but TUNE only and MODEL→TUNE miss the CPU floor; the changed outcomes prevent a general recipe ranking. The frozen decoder and MODEL were never trained on the terse TUNE template and frequently emit prose rather than a parseable number, as the coverage entries in Tab. 27 show.