Jacobian Sparse Autoencoders: Sparsify Computations, Not Just Activations

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Abstract

Sparse autoencoders (SAEs) have been successfully used to discover sparse and human-interpretable representations of the latent activations of LLMs. However, we would ultimately like to understand the computations performed by LLMs and not just their representations. The extent to which SAEs can help us understand computations is unclear because they are not designed to "sparsify" computations in any sense, only latent activations. To solve this, we propose Jacobian SAEs (JSAEs), which yield not only sparsity in the input and output activations of a given model component but also sparsity in the computation (formally, the Jacobian) connecting them. With a naïve implementation, the Jacobians in LLMs would be computationally intractable due to their size. One key technical contribution is thus finding an efficient way of computing Jacobians in this setup. We find that JSAEs extract a relatively large degree of computational sparsity while preserving downstream LLM performance approximately as well as traditional SAEs. We also show that Jacobians are a reasonable proxy for computational sparsity because MLPs are approximately linear when rewritten in the JSAE basis. Lastly, we show that JSAEs achieve a greater degree of computational sparsity on pre-trained LLMs than on the equivalent randomized LLM. This shows that the sparsity of the computational graph appears to be a property that LLMs learn through training, and suggests that JSAEs might be more suitable for understanding learned transformer computations than standard SAEs.

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1. Introduction

Sparse autoencoders (SAEs) have emerged as a powerful tool for understanding the internal representations of large language models (Bricken et al., 2023; Cunningham et al., 2023; Gao et al., 2024; Rajamanoharan et al., 2024b; Lieberum et al., 2024; Lawson et al., 2024; Braun et al., 2024; Kissane et al., 2024; Rajamanoharan et al., 2024a). By decomposing neural network activations into sparse, interpretable components, SAEs have helped researchers gain significant insights into how these models process information (Marks et al., 2024; Lieberum et al., 2024; Templeton et al., 2024b; O'Brien et al., 2024; Farrell et al., 2024; Paulo et al., 2024; Balcells et al., 2024; Lan et al., 2024; Brinkmann et al., 2025; Spies et al., 2024).

When trained on the activation vectors from neural network layers, SAEs learn to reconstruct the inputs using a dictionary of sparse 'features', where there are many more features than basis dimensions of the inputs, and each feature tends to capture a specific, interpretable concept. However, the goal of this paper is to improve understanding of *computations* in transformers. While SAEs are designed to disentangle the representations of concepts in the LLM, they are not designed to help us understand the computations performed with those representations.

One approach to understanding computation would be to train two SAEs, one at the input and one at the output of an MLP in a transformer. We can then ask how the MLP maps sparse latent features at the inputs to sparse features in the outputs. For this mapping to be interpretable, it would be desirable that it is sparse, in the sense that each latent in the SAE trained on the output depends on a small number of latents of the SAE trained on the input. These dependencies can be understood as a computation graph or 'circuit' (Olah et al., 2020; Cammarata et al., 2020). SAEs are not designed to encourage this computation graph to be sparse. To address this, we develop Jacobian SAEs (JSAEs), where we include a term in the objective to encourage SAE bases with sparse computational graphs, not just sparse activations. Specifically, we treat the mapping between the latent activations of the input and output SAEs as a function and encourage its Jacobian to be sparse by including an L^1 penalty term in the loss function.

With a naïve implementation, it is intractable to compute

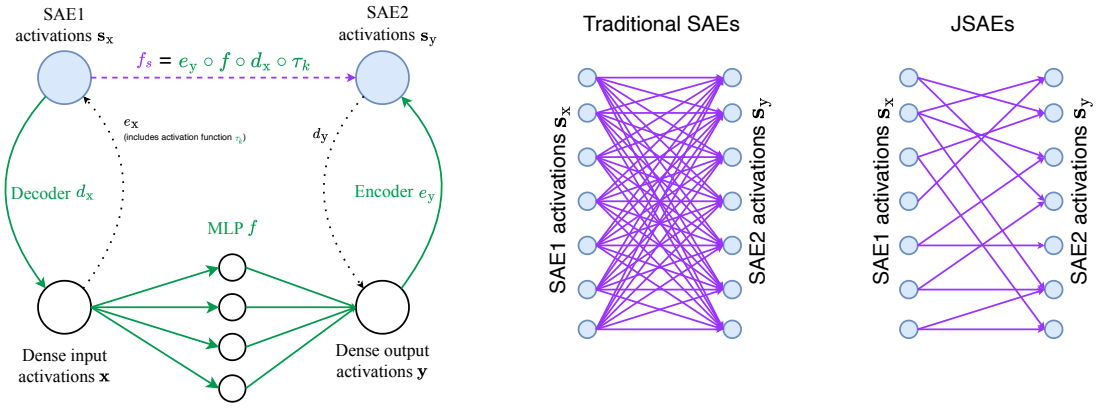

Figure 1. A diagram illustrating our setup. We have two SAEs: one trained on the MLP inputs and the other trained on the MLP outputs. We then consider the function f_s, which takes the latent activations of the first SAE and returns the latent activations of the second SAE, i.e., $f_s(\mathbf{s}_x) = \mathbf{s}_y$. The function f_s is described by the function composition of the TopK activation function of the first (input) SAE τ_k, the decoder of the first SAE d_x, the MLP f, and the encoder of the second (output) SAE e_y. We note that the activation function τ_k is included for computational efficiency only; see Section 4.2 for details. JSAEs optimize for f_s having a sparse Jacobian matrix, which we illustrate by reducing the number of edges in the computational graph that corresponds to f_s. Traditional SAEs have sparse SAE latents on either side of the MLP but a dense computational graph between them; JSAEs have both sparse SAE latents *and* a sparse computational graph. Importantly, Jacobian sparsity approximates the computational graph notion, but, as we discuss in Section 5.4 and Appendix B, this approximation is highly accurate due to the fact that f_s is a mostly linear function.

Jacobian matrices because each matrix would have on the order of a trillion elements, even for modestly sized language models and SAEs. Therefore, one of our core contributions is to develop an efficient means to compute Jacobian matrices in this context. The approach we develop makes it possible to train a pair of Jacobian SAEs with only approximately double the computational requirements of training a single standard SAE (Section 4.2). These methods enabled us to make three downstream findings.

First, we find that Jacobian SAEs successfully induce sparsity in the Jacobian matrices between input and output SAE latents relative to standard SAEs without a Jacobian term (Section 5.1). We find that JSAEs achieve the desired increase in the sparsity of the Jacobian with only a slight decrease in reconstruction quality and model performance preservation, which remain roughly on par with standard SAEs. We also find that the input and output latents learned by Jacobian SAEs are approximately as interpretable as standard SAEs, as quantified by auto-interpretability scores. Importantly, we also find that the "computational units" discovered by JSAEs are often highly interpretable – for example, JSAEs find an output latent corresponding to whether the text is in German, which is computed using several input latents corresponding to tokens frequently found in German text (Section 5.2).

Second, inspired by Heap et al. (2025), we investigated the behavior of Jacobian SAEs when applied to random transformers, i.e., where the parameters have been reinitialized. We find that the degree of Jacobian sparsity that can be achieved when JSAEs are applied to a pre-trained transformer is much greater than the sparsity achieved for a random transformer (Section 5.3). This preliminary finding suggests that Jacobian sparsity may be a useful tool for discovering learned computational structure.

Lastly, we find that Jacobians accurately approximate computational sparsity in this context because the function we are analyzing (i.e., the combination of JSAEs and MLP) is approximately linear (Section 5.4).

Our source code can be found at https://github.com/lucyfarnik/jacobian-saes.

2. Related work

2.1. Sparse autoencoders

SAEs have been widely applied to 'disentangle' the representations learned by transformer language models into a very large number of concepts, a.k.a. sparse latents, features, or dictionary elements (Sharkey et al., 2022; Cunningham et al., 2023; Bricken et al., 2023; Gao et al., 2024; Rajamanoharan et al., 2024b; Lieberum et al., 2024). Human experiments and quantitative proxies apparently confirm that SAE latents are much more likely to correspond to human-interpretable concepts than raw language-model neurons,

i.e., the basis dimensions of their activation vectors (Cunningham et al., 2023; Bricken et al., 2023; Rajamanoharan et al., 2024a). SAEs have been successfully applied to modifying the behavior of LLMs by using a direction discovered by an SAE to "steer" the model towards a certain concept (Makelov, 2024; O'Brien et al., 2024; Templeton et al., 2024b).

Our work is based on SAEs but has a very different aim: standard SAEs only sparsify activations, while JSAEs also sparsify the computation graph between them (Figure 1).

2.2. Transcoders

In this paper, we focus on MLPs. Dunefsky et al. (2024); Templeton et al. (2024a) developed *transcoders*, an alternative SAE-like method to understand MLPs. However, JSAEs and transcoders take radically different approaches and solve radically different problems. This is perhaps easiest to see if we look at what transcoders and JSAEs sparsify. JSAEs are fundamentally an extension of standard SAEs: they train SAEs at the input and output of the MLP and add an extra term to the objective such that these sparse latents are also appropriate for interpreting the MLP (Figure 1). In contrast, transcoders do not sparsify the inputs and outputs; they work with dense inputs and outputs. Instead, transcoders, in essence, sparsify the MLP hidden states. Specifically, a transcoder is an MLP that you train to match (using a mean squared error objective) the input-to-output mapping of the underlying MLP from the transformer. The key difference between the transcoder MLP and the underlying MLP is that the transcoder MLP is much wider, and its hidden layer is trained to be sparse.

Thus, transcoders and JSAEs take fundamentally different approaches. Each transcoder latent tells us 'there is computation in the MLP related to [concept].' By comparison, JSAEs learn a pair of SAEs (which have mostly interpretable latents) and sparse connections between them. At a conceptual level, JSAEs tell us that 'this feature in the MLP's output was computed using only these few input features'. Ultimately, we believe that the JSAE approach, grounded in understanding how the SAE basis at one layer is mapped to the SAE basis at another layer, is potentially powerful and worth thoroughly exploring.

Importantly, it is worth emphasizing that JSAEs and transcoders are asking fundamentally different questions, as can be seen in terms of e.g., differences in what they sparsify. As such, it is not, to our knowledge, possible to design meaningful quantitative comparisons, at least not without extensive future work to develop very general auto-interpretability methods for evaluating methods of understanding MLP circuits.

2.3. Automated circuit discovery

In "automated circuit discovery", the goal is to isolate the causally relevant intermediate variables and connections between them necessary for a neural network to perform a given task (Olah et al., 2020). In this context, a circuit is defined as a computational subgraph with an interpretable function. The causal connections between elements are determined via activation patching, i.e., modifying or replacing the activations at a particular site of the model (Meng et al., 2022; Zhang & Nanda, 2023; Wang et al., 2022; Hanna et al., 2023). In some cases, researchers have identified subcomponents of transformer language models with simple algorithmic roles that appear to generalize across models (Olsson et al., 2022).

Conmy et al. (2023) proposed a means to automatically prune the connections between the sub-components of a neural network to the most relevant for a given task using activation patching. Given a choice of task (i.e., a dataset and evaluation metric), this approach to automated circuit discovery (ACDC) returns a minimal computational subgraph needed to implement the task, e.g., previously identified 'circuits' like Hanna et al. (2023). Naturally, this is computationally expensive, leading other authors to explore using linear approximations to activation patching (Nanda, 2023; Syed et al., 2024; Kramár et al., 2024). Marks et al. (2024) later improved on this technique by using SAE latents as the nodes in the computational graph.

In a sense, these methods are supervised because they require the user to specify a task. Naturally, it is not feasible to manually iterate over all tasks an LLM can perform, so a fully unsupervised approach is desirable. With JSAEs, we take a step towards resolving this problem, although the architecture introduced in this paper initially only applies to a single MLP layer and not an entire model. Additionally, to the best of our knowledge, no automated circuit discovery algorithm sparsifies the computations inside of MLPs.

There are also other approaches which focus on locating relevant computation in ML models by estimating the contribution of individual model components (Shah et al., 2024; Balasubramanian et al., 2024).

3. Background

3.1. Sparse autoencoders

In an SAE, we have input vectors, $\mathbf{x} \in \mathcal{X} = \mathbb{R}^{m_\text{x}}$. We want to approximate each vector $\mathbf{x}$ by a sparse linear combination of vectors, $\mathbf{s}_\text{x} \in \mathcal{S}_\text{x} = \mathbb{R}^{n_\text{x}}$. The dimension of the sparse vector, n_x, is typically much larger than the dimension of the input vectors m_x (i.e. the basis is overcomplete).

In the case of SAEs, we treat the vectors as inputs to an autoencoder with an encoder $e_\text{x} : \mathcal{X} \to \mathcal{S}_\text{x}$ and a decoder

$d_{\text{x}} : \mathcal{S}_{\text{x}} \to \mathcal{X}$ defined by,

$$\mathbf{s}_{\text{x}} = e_{\text{x}}(\mathbf{x}) = \phi(\mathbf{W}_{\text{x}}^{\text{enc}}\mathbf{x} + \mathbf{b}_{\text{x}}^{\text{enc}}) \quad (1)$$

$$\hat{\mathbf{x}} = d_{\text{x}}(\mathbf{s}_{\text{x}}) = \mathbf{W}_{\text{x}}^{\text{dec}}\mathbf{s}_{\text{x}} + \mathbf{b}_{\text{x}}^{\text{dec}} \quad (2)$$

Here, the parameters are the encoder weights $\mathbf{W}_{\text{enc}} \in \mathbb{R}^{n_{\text{x}} \times m_{\text{x}}}$, decoder weights $\mathbf{W}_{\text{dec}} \in \mathbb{R}^{m_{\text{x}} \times n_{\text{x}}}$, encoder bias $\mathbf{b}_{\text{x}}^{\text{enc}} \in \mathbb{R}^{n_{\text{x}}}$, and decoder bias $\mathbf{b}_{\text{x}}^{\text{dec}} \in \mathbb{R}^{m_{\text{x}}}$. The non-linearity ϕ can be, for instance, ReLU. These parameters are then optimized to minimize the difference between $\mathbf{x}$ and $\hat{\mathbf{x}}$, typically measured in terms of the mean squared error (MSE), while imposing an L^1 penalty on the latent activations $\mathbf{s}_{\text{x}}$ to incentivize sparsity.

3.2. Automatic interpretability of SAE latents

In order to compare the quality of different SAEs, it is desirable to be able to quantify how interpretable its latents are. A popular approach to quantifying interpretability at scale is to collect the examples that maximally activate a given latent, prompt an LLM to generate an explanation of the concept the examples have in common, and then prompt an LLM to predict whether a given prompt activates the SAE latent given the generated explanation. We can then score the accuracy of the predicted activations relative to the ground truth. There are several variants of this approach (e.g., Bills et al., 2023; Choi et al., 2024); in this paper, we use "fuzzing" where the scoring model classifies whether the highlighted tokens in prompts activate an SAE latent given an explanation of that latent (Paulo et al., 2024).

4. Methods

The key idea with a Jacobian SAE is to train a pair of SAEs on the inputs and outputs of a neural network layer while additionally optimizing the sparsity of the Jacobian of the function that relates the input and output SAE latent activations (Figure 1). In this paper, we apply Jacobian SAEs to multi-layer perceptrons (MLPs) of the kind commonly found in transformer language models (Radford et al., 2019; Biderman et al., 2023).

4.1. Setup

Consider an MLP mapping from $\mathbf{x} \in \mathcal{X}$ to $\mathbf{y} \in \mathcal{Y}$, i.e., $f : \mathcal{X} \to \mathcal{Y}$ or $\mathbf{y} = f(\mathbf{x})$. We can then train two k-sparse SAEs, one on $\mathbf{x}$ and the other on $\mathbf{y}$. The resulting SAEs map from each of $\mathbf{x}$ and $\mathbf{y}$ to corresponding sparse latents $\mathbf{s}_{\text{x}} \in \mathcal{S}_{\text{x}}$ and $\mathbf{s}_{\text{y}} \in \mathcal{S}_{\text{y}}$, i.e., $\mathbf{s}_{\text{x}} = e_{\text{x}}(\mathbf{x})$ and $\mathbf{s}_{\text{y}} = e_{\text{y}}(\mathbf{y})$, where e_{x} is the encoder of the first SAE and e_{y} is the encoder of the second SAE. Each of these SAEs also has a decoder that maps from the sparse latents back to an approximation of the original vector: $\hat{\mathbf{x}} = d_{\text{x}}(\mathbf{s}_{\text{x}})$ and $\hat{\mathbf{y}} = d_{\text{y}}(\mathbf{s}_{\text{y}})$.

We may now consider the function $f_s : S_X \to S_Y$, which intuitively represents the function, f, but written in terms of the sparse bases learned by the SAE pair for the original vectors $\mathbf{x}$ and $\mathbf{y}$. Specifically, we define f_s by

$$f_s = e_{\text{y}} \circ f \circ d_{\text{x}} \circ \tau_k \quad (3)$$

where $\circ$ denotes function composition. Here, $d_{\text{x}} : \mathcal{S}_{\text{x}} \to \mathcal{X}$ maps the sparse latents given as input to f_s to "dense" inputs. Then, $f : \mathcal{X} \to \mathcal{Y}$ maps the dense inputs to dense outputs. Finally, $e_{\text{y}} : \mathcal{Y} \to \mathcal{S}_{\text{y}}$ maps the dense outputs to sparse outputs. Note that f_s first applies the TopK activation function τ_k to the sparse inputs, $\mathbf{s}_{\text{x}}$. Critically, with k-sparse SAEs, we produce the sparse inputs by $\mathbf{s}_{\text{x}} = e_{\text{x}}(\mathbf{x})$, implying that $\mathbf{s}_{\text{x}}$ only has k non-zero elements. In that setting, TopK does not change the inputs, i.e. $\mathbf{s}_{\text{x}} = \tau_k(\mathbf{s}_{\text{x}})$, but it does affect the Jacobian and, in particular, allows us to compute it much more efficiently (Section 4.2).

At a high level, we want the function f_s to be 'sparse', in the sense that each of its input dimensions (i.e. SAE latent activations) only affects a small number of its output dimensions, and each of its output dimensions only depends on a small number of its input dimensions. We quantify the sparsity of f_s in terms of its Jacobian matrix. The Jacobian of f_s is, in index notation:

$$J_{f_s,i,j} = \frac{\partial f_{s,i}(s_{\text{x}})}{\partial s_{\text{x},j}}. \quad (4)$$

Intuitively, we can consider maximizing the sparsity of the Jacobian as minimizing the number of edges in the computational graph connecting the input and output nodes (Figure 1), i.e. maximizing the number of near-zero elements in the Jacobian matrix. We note that the Jacobian is not a perfect measure of the sparsity of the computational graph, but it is an accurate proxy (see Section 5.4 and Appendix B) while being computationally tractable.

We simultaneously train two separate SAEs on the input and output of a transformer MLP with the objectives of low reconstruction error and sparse relations between the separate SAE latents (via the Jacobian). We do not need to optimize for the sparsity of the latent activations via a penalty term in the loss function because we use k-sparse autoencoders, which keep only the k largest latent activations per token position. Hence, our loss function is

$$\mathcal{L} = \text{MSE}(\mathbf{x}, \hat{\mathbf{x}}) + \text{MSE}(\mathbf{y}, \hat{\mathbf{y}}) + \frac{\lambda}{k^2} \sum_{i=1}^{n_{\text{y}}} \sum_{j=1}^{n_{\text{x}}} |J_{f_s,i,j}| \quad (5)$$

Here, k is the number of non-zero elements in the TopK activation function, $n_{\text{x}}, n_{\text{y}}$ are the dimensionalities of the latent spaces of the input and output SAEs, respectively, and λ is the coefficient of the Jacobian loss term. We divide by k^2 because, as we will see later, there are at most k^2 non-zero elements in the Jacobian. Finally, note that if we set $\lambda = 0$, then our objective effectively trains traditional SAEs for each of $\mathbf{x}$ and $\mathbf{y}$ independently.

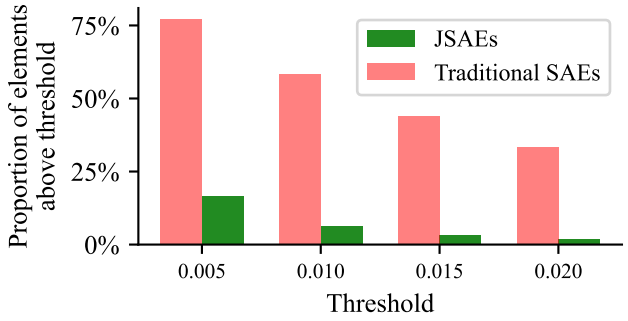

Figure 2. JSAEs induce a much greater degree of sparsity in the elements of the Jacobian of f_s than traditional SAEs. The bars show the average proportion of Jacobian elements with absolute values above certain thresholds. At most $k \times k$ elements can be nonzero, so we take 100% on the y-axis to mean $k \times k$. The average was taken across 10 million tokens. This example is from layer 15 of Pythia-410m. For layer 3 of Pythia-70m and layer 7 of Pythia-160m, see Figure 34, for more quantitative information on Jacobian sparsity across model sizes, layers, and hyperparameters see Figures 24, 25, and 26. We present further discussion of the sparsity of the Jacobian in Appendix F.

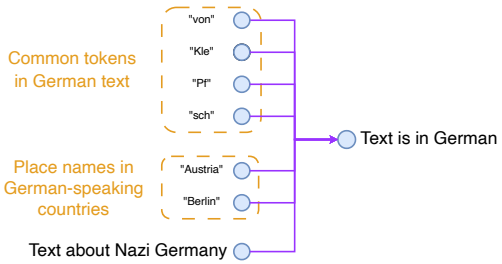

Figure 3. JSAEs allow us to locate the "input features" of each feature computed by the MLP. For instance, in Pythia-410m, the MLP at layer 15 is computing the feature "this text is in German". JSAEs discover the inputs which the MLP uses to decide whether this feature should be on or off. These inputs correspond to tokens frequently found in German text, place names in German-speaking countries, and text about Nazi Germany. See Appendix C for details.

4.2. Making the Jacobian calculation tractable

Computing the Jacobian naively (e.g., using an automatic differentiation package) is computationally intractable, as the full Jacobian has size $B \times n_y \times n_x$ where B is the number of tokens in a training batch n_x is the number of SAE latents for the input, and n_y is the number of SAE latents for the output. Unfortunately, typical values are around $1,000$ for B and around $32,000$ for n_x and n_y (taking as an example a model dimension of $1,000$ and an expansion factor of 32). Combined, this gives a Jacobian with around 1 trillion elements. This is obviously far too large to work with in practice, and our key technical contribution is to develop an efficient approach to working with this huge Jacobian.

Our first insight is that for each element of the batch, we have a $n_y \times n_x$ Jacobian, where n_x and n_y are around $32,000$. This is obviously far too large. However, remember that we are interested in the Jacobian of f_s, so the input is the sparse SAE latent vector, $\mathbf{s}_x$ and the output is the sparse SAE latent vector, $\mathbf{s}_y$. Importantly, as we are using k-sparse SAEs, only k elements of the input and output are "on" for any given token. As such, we really only care about the $k \times k$ elements of the Jacobian of f_s, corresponding to the inputs and outputs that are "on". This reduces the size of the Jacobian by around six orders of magnitude, and renders the computation tractable. However, to make this work formally, we need all elements of the Jacobian corresponding to "off" elements of the input and output to be zero. This is where the τ_k in the definition of f_s becomes important. Specifically, the τ_k ensures that the gradient of f_s wrt any of the inputs that are "off" is zero. Without τ_k, the Jacobian could be non-zero for any of the inputs, even if changing those inputs would not make sense, as it would give more than k elements being "on" in the input, and thus could not be produced by the k-sparse SAE.

Our second insight was that computing the Jacobian by automatic differentiation would still be relatively inefficient, e.g., requiring k backward passes. Instead, for standard GPT-2-style MLPs, we noticed that an extremely efficient Jacobian formula can be derived by hand, requiring only three matrix multiplications and along with a few pointwise operations. We present this derivation in Appendix A.

With these optimizations in place, training a pair of JSAEs takes about twice as long as training a single standard SAE. We measured this by training ten of each model on Pythia-70m with an expansion factor of 32 for 100 million tokens on an RTX 3090. The average training durations were 72mins for a pair of JSAEs and 33 mins for a traditional SAE, with standard deviations below 30 seconds for both.

5. Results

Our experiments were performed on LLMs from the Pythia suite (Biderman et al., 2023), the figures in the main text contain results from Pythia-410m unless otherwise specified. We trained on 300 million tokens with $k = 32$ and an expansion factor of 64 for Pythia-410m and 32 for smaller models. We reproduced all our experiments on multiple models and found the same qualitative results (see Appendix E).

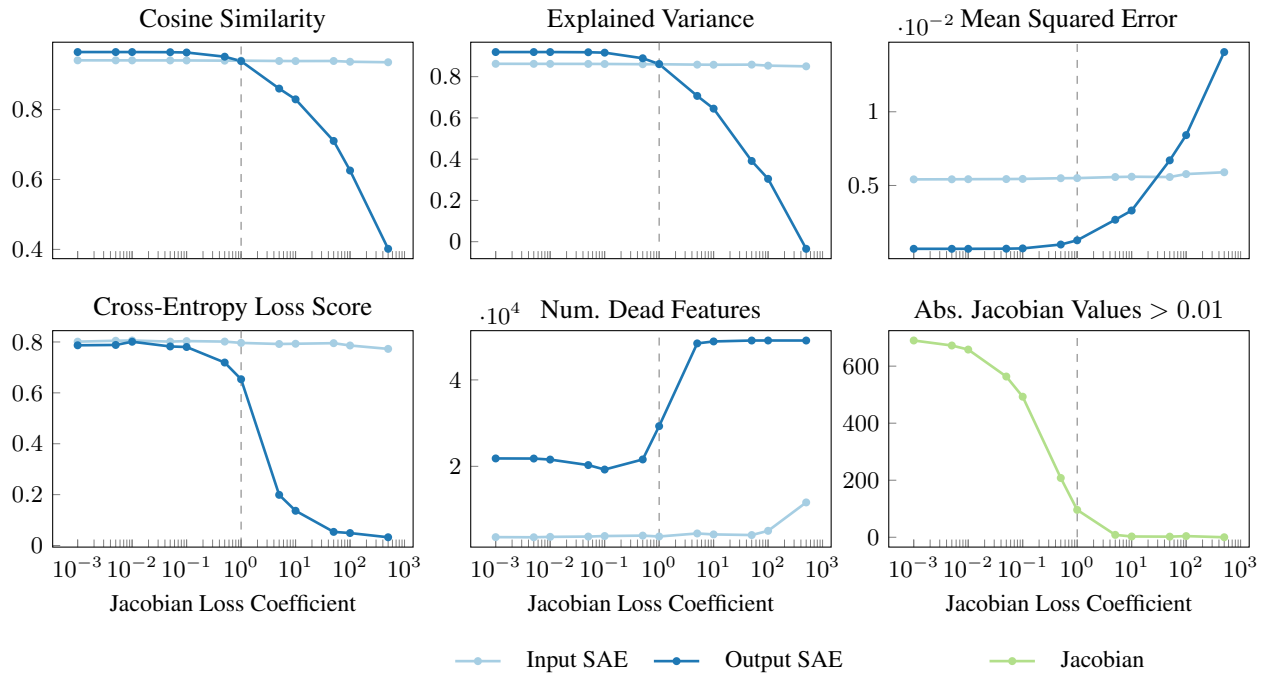

Figure 4. Reconstruction quality, model performance preservation, and sparsity metrics against the Jacobian loss coefficient. JSAEs trained on layer 7 of Pythia-160m with expansion factor 64 and $k = 32$; see Figure 26 for layer 3 of Pythia-70m. Recall that the maximum number of non-zero Jacobian values is $k^2 = 1024$. In accordance with Figure 5, all evaluation metrics degrade for values of the coefficient above 1. See Appendix E for details of the evaluation metrics.

5.1. Jacobian sparsity, reconstruction quality, and auto-interpretability scores

First, we compared the Jacobian sparsity for standard SAEs and JSAEs. Note that, unlike with SAE latent activations, there is no mechanism for producing exact zeros in the Jacobian elements corresponding to active latents. Hence, we consider the number of near-zero elements rather than the number of exact zeros. To quantify the difference in sparsity between the two, we looked at the proportion of the elements of the Jacobian above a particular threshold when aggregating over 10 million tokens (Figure 2). Here, we found that JSAEs dramatically reduced the number of large elements of the Jacobian relative to traditional SAEs. We also note that the Jacobians are not only sparse on each individual token, but also when averaged across a large number of tokens (see Figure 36 in the appendix).

Importantly, the degree of sparsity depends on our choice of the coefficient λ of the Jacobian loss term. Therefore, we trained multiple JSAEs with different values of this parameter. As we might expect, for small values of λ, i.e., little incentive to sparsify the Jacobian, the input and output SAEs perform similarly to standard SAEs (Figure 4 blue lines), including in terms of the variance explained by the reconstructed activation vectors and the increase in the cross-entropy loss when the input activations are replaced by their reconstructions. Unsurprisingly, as λ grows larger and the Jacobian loss term starts to dominate, our evaluation metrics degrade. Interestingly, this degradation happens almost entirely in the output SAE rather than the input SAE — we leave it to future work to investigate this phenomenon further.

Critically, Figure 4 suggests there is a 'sweet spot' of the λ hyperparameter where the SAE quality metrics remain reasonable, but the Jacobian is much sparser than for standard SAEs. To further investigate this trade-off, we plotted a measure of Jacobian sparsity (the proportion of elements of the Jacobian above 0.01) against the average cross-entropy (Figures 4, 5, and 29). We found that there is indeed a sweet spot where the average cross-entropy is only slightly worse than a traditional SAE, while the Jacobian is far sparser. For Pythia 410m (Figure 5) this value is around $\lambda = 0.5$, whereas for Pythia-70m, it is around $\lambda = 1$ (Figure 29). We choose this value of the Jacobian coefficient (i.e. $\lambda = 0.5$ for Pythia-410m in the main text, and $\lambda = 1$ for Pythia-160m in the Appendix) in other experiments.

We also measure the interpretability of JSAE latents using the automatic interpretability pipeline developed by Paulo et al. (2024) and compare this to traditional SAEs. We find that JSAEs achieve similar interpretability scores (Figure 6).

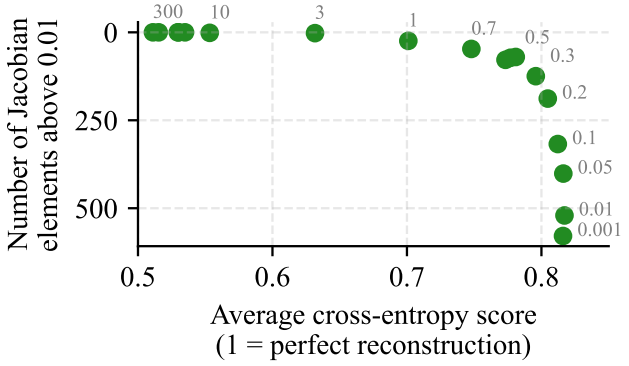

Figure 5. The trade-off between reconstruction quality and Jacobian sparsity as we vary the Jacobian loss coefficient. Each dot represents a pair of JSAEs trained with a specific Jacobian coefficient. The value of λ is included for some points. We can see that a coefficient of roughly $\lambda = 0.5$ is optimal for Pythia-410m with $k = 32$. Note that the CE loss score is the average of the CE loss scores of the pre-MLP JSAE and the post-mlp JSAE. Measured on layer 15 of Pythia-410m, similar charts with a wider range of models and metrics can be found in Figures 27, 28, and 29.

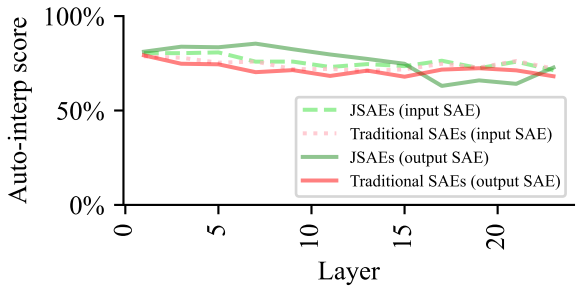

Figure 6. Automatic interpretability scores of JSAEs are very similar to traditional SAEs. Measured on all odd-numbered layers of Pythia-410m using the "fuzzing" scorer from Paulo et al. (2024). For all layers of Pythia-70m see Figure 37.

5.2. Max-activating examples of JSAEs

Next, we interpreted the "max-activating" examples of JSAEs in order to verify that JSAEs can locate semantically meaningful computational units. Namely, we took the latents of the output SAE i which have large Jacobian values when averaging across a wide distribution of text. Then for each output SAE latent i, we found the 10 input SAE latents j which have the largest average Jacobian elements $J_{f_s,i,j}$. We find that these combinations are often highly interpretable. For example, as shown in Figure 3, the very first output latent of layer 15 of Pythia-410m as sorted by average Jacobian value corresponds to "this text is in German". We find that it is computed as a function of input latents corresponding to:

- Tokens which frequently appear in German text, such as "Pf", "sch", "Kle", and "von"
- Names of places where people speak German, such as "Berlin" or "Austria"
- Words and phrases related to the Third Reich, such as "Nazi", "concentration camp", "Hitler", and "Holocaust"

For a few of handpicked examples, see Appendix C. A large number of examples which are not handpicked is available at tinyurl.com/jsaes-qualitative.

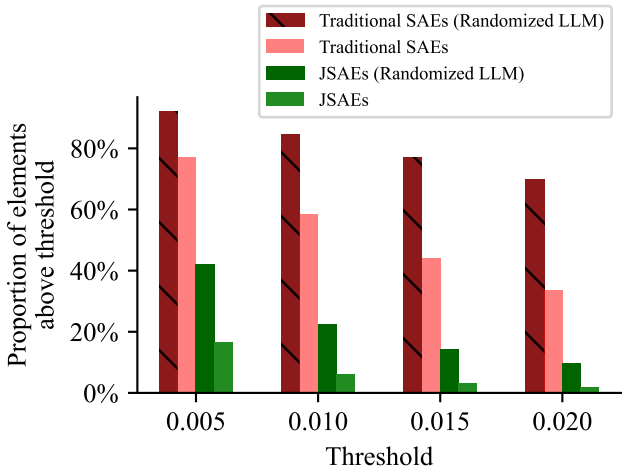

Figure 7. Jacobians are substantially more sparse in pre-trained LLMs than in randomly initialzied transformers. This holds both when you actively optimize for Jacobian sparsity with JSAEs, and when you don't optimize for it and use traditional SAEs. The figure shows the proportion of Jacobian elements with absolute values above certain thresholds. At most k^2 elements can be nonzero, we therefore take k^2 to be 100% on the y-axis. Jacobians are significantly more sparse in pre-trained transformers than in randomly re-initialized transformers. This shows that Jacobian sparsity is, at least to some extent, connected to the structures that LLMs learn during training. This stands in contrast to recent work by Heap et al. (2025) showing that traditional SAEs achieve roughly equal auto-interpretability scores on randomly initialized transformers as they do on pre-trained LLMs. Measured on layer 15 of Pythia-410m, for layer 3 of Pythia-70m see Figure 38. Averaged across 10 million tokens.

5.3. Performance on re-initialized transformers

To confirm that JSAEs are extracting information about the complex learned computation, we considered a form of control analysis inspired by Heap et al. (2025). Specifically, we would expect that trained transformers have carefully learned specific, structured computations while randomly initialized transformers do not. Thus, a possible desideratum for tools in mechanistic interpretability is that they

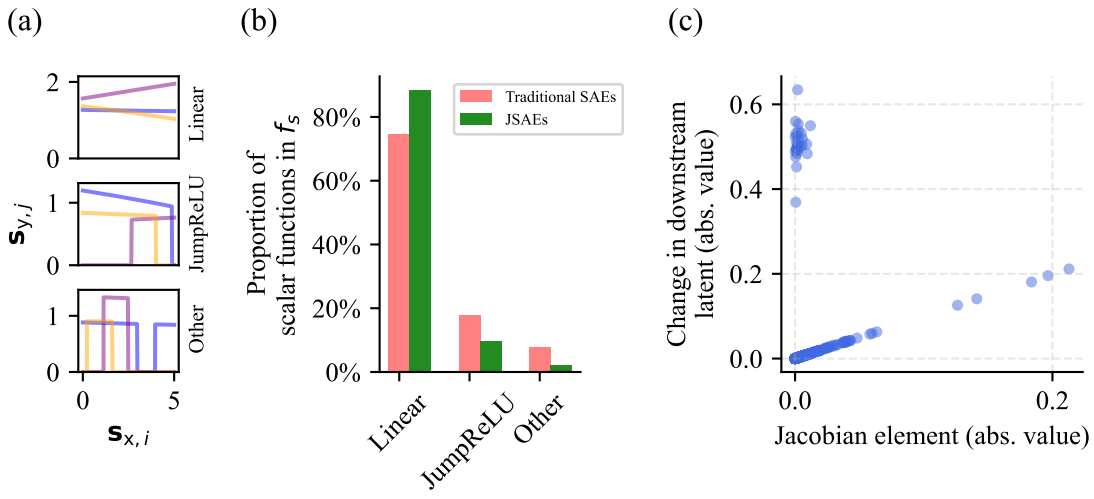

Figure 8. The function f_s, which combines the decoder of the first SAE, the MLP, and the encoder of the second SAE, is mostly linear. Specifically, the vast majority of scalar functions going from $s_{\mathrm{x},j}$ to $s_{\mathrm{y},i}$ are linear. (a) Examples of linear, JumpReLU, and other functions relating individual input SAE latents and output SAE latents. See Figure 9 for more examples. (b) For the empirically observed $\mathbf{s}_{\mathrm{x}}$ and randomly selected i, j (of those corresponding to active SAE latents), the vast majority of scalar functions from $s_{\mathrm{x},j}$ to $s_{\mathrm{y},i}$ are linear. For details see Appendix B. The proportion of linear function also noticeably increases with JSAEs compared to traditional SAEs, meaning that JSAEs induce additional linearity in f_s. (c) Because the vast majority of functions are linear, the Jacobian usually precisely predicts the change observed in the output SAE latent when we make a large change to the input SAE latent's value (namely subtracting 1, note that the empirical median value of $s_{\mathrm{x},j}$ is 2.5). Each dot corresponds to an $(s_{\mathrm{x},j}, s_{\mathrm{y},i})$ pair. For 97.7% of pairs (across a sample size of 10 million) their Jacobian value nearly exactly predicts the change we see in the output SAE latent when making large changes to the input SAE latent's activation, i.e. $|\Delta s_{\mathrm{y},i}| \approx |J_{f_s,ij}|$. The scatter plot shows a randomly selected subset of 1,000 $(s_{\mathrm{x},j}, s_{\mathrm{y},i})$ pairs. For further details see Appendix B. Measured on layer 15 of Pythia-410m, for layer 3 of Pythia-70m see Figure 39, for the linearity results on other models and hyperparameters see Figures 15, 16, and 17.

ought to work substantially better when analyzing the complex computations in trained LLMs than when applied to LLMs with randomly re-initialized weights. This is precisely what we find. Specifically, we find that the Jacobians for trained networks are always substantially sparser than the corresponding random trained network, and this holds for both traditional SAEs and JSAEs (Figure 7). Further, the relative improvement in sparsity from the traditional SAE to the JSAE is much larger for trained than random LLMs, again indicating that JSAEs are extracting structure that only exists in the trained network. Note that we also see that for traditional SAEs, there is a somewhat more sparse Jacobian for the trained than randomly initialized transformer. This makes sense: we would hope that the traditional SAE basis is somewhat more aligned with the computation (as expressed by a sparse Jacobian) than we would expect by chance. However, it turns out that without a "helping hand" from the Jacobian sparsity term, the alignment in a traditional SAE is relatively small. Thus, Jacobian sparsity is a property related to the complex computations LLMs learn during training, which should make it substantially useful for discovering the learned structures of LLMs.

5.4. f_s is mostly linear

Importantly, the Jacobian is a local measure. Thus, strictly speaking, a near-zero element of the Jacobian matrix implies only that a small change to the input SAE latent does not affect the corresponding output SAE latent. It may, however, still be the case that a large change to the input SAE latent would change the output SAE latent. We investigated this question and found that f_s is usually approximately linear in a wide range and is often close to linear. Specifically, of the scalar functions relating individual input SAE latents $s_{\mathrm{x},j}$ to individual output SAE latents $s_{\mathrm{y},i}$, the vast majority are linear (Figure 8b). This is important because, for any linear function, its local slope is completely predictive of its global shape, and therefore, a near-zero Jacobian element implies a near-zero causal relationship. For the scalar functions which are not linear, we frequently observed they have a JumpReLU structure[1] (Erichson et al., 2019). Notably, a JumpReLU is linear in a subset of its input space, so even for these scalar functions the first derivative is still an accurate measure within some range of $s_{\mathrm{x},j}$ values. It is also worth

[1]By JumpReLU, we mean any function of the form $f(x) = a\mathrm{JumpReLU}(bx + c)$. Recall that $\mathrm{JumpReLU}(x) = x$ if $x > d$ and 0 otherwise. $a, b, c, d \in \mathbb{R}$ are constants.

noting that with JSAEs, the proportion of linear functions is noticeably higher than with traditional SAEs, so at least to a certain extent, JSAEs induce additional linearity in the MLP. To confirm these results, we plotted the Jacobian against the change of output SAE latent $s_{\mathrm{y},i}$ as we change the input SAE latent $s_{\mathrm{x},j}$ by subtracting 1 (Figure 8c)[2]. We found that 97.7% of the time, $|\Delta s_{\mathrm{y},i}| \approx |J_{f_s,ij}|$. For details see Appendix B. While these results are strongly suggestive, we would caution that it is difficult to interpret them definitively as we are not evaluating the reconstruction error for a linear model fitted to the input-output relationship for the MLP latents.

6. Discussion

We believe JSAEs are a promising approach for discovering computational sparsity and understanding the reasoning of LLMs. We would also argue that an approach like the one we introduced is in some sense necessary if we want to 'reverse-engineer' or 'decompile' LLMs into readable source code. It is not enough that our variables (e.g., SAE features) are interpretable; they must also be connected in a relatively sparse way. To illustrate this point, imagine a Python function that takes as input 5 arguments and returns a single variable, and compare this to a Python function that takes 32,000 arguments. Naturally, the latter would be nearly impossible to reason about. Discovering computational sparsity thus appears to be a prerequisite for solving interpretability. It is also important that the mechanisms for discovering computational sparsity be fully unsupervised rather than requiring the user to manually specify the task being analyzed. There are existing methods for taking a specific task and finding the circuit responsible for implementing it, but these require the user to specify the task first (e.g. as a small dataset of task-relevant prompts and a metric of success). They are thus 'supervised' in the sense that they need a clear direction from the user. Naturally, it is not feasible to manually iterate over all tasks an LLM may be performing, so a fully unsupervised approach is needed. JSAEs are the first step in this direction.

Naturally, JSAEs in their current form still have important limitations. They currently only work on MLPs, and for now, they only operate on a single layer at a time rather than discovering circuits throughout the entire model. Our initial implementation also works on GPT-2-style MLPs, while most LLMs from the last few years tend to use GLUs (Dauphin et al., 2017; Shazeer, 2020), though we expect it to be fairly easy to extend our setup to GLUs. Additionally, our current implementation relies on the TopK activation function for efficient batching; TopK SAEs can sometimes encourage high-density features, so it may be desirable to generalize our implementation to work with other activation functions. These are, however, problems that can be addressed relatively straightforwardly in future work, and we would welcome correspondence from researchers interested in addressing them.

A pessimist may argue that partial derivatives (and, therefore, Jacobians) are merely local measures. A small partial derivative tells you that if you slightly tweak the input latent's activation, you will see no change to the output latent's activation, but it may well be the case that a large change to the input latent's activation will lead to a large change in the output latent. Thankfully, at least in MLPs, this is not quite the case. As we show in Section 5.4, f_s is approximately linear, and the size of the elements of the Jacobian nearly perfectly predicts the change you see in the output latent when you make a large change to the input latent. For a linear function, a first-order derivative at any point is perfectly predictive of the relationship between the input and the output, and thus, at least for the fraction of f_s that is linear, Jacobians perfectly measure the computational relationship between input and output variables. We further discuss this in Appendix B. Additionally, as we showed in Section 5.3, Jacobian sparsity is much more present in trained LLMs than in randomly initialized ones, which indicates that it does correspond in some way to structures that were learned during training. At a high level, a sparse computational graph necessarily implies a sparse Jacobian, but a sparse Jacobian does not in and of itself imply a sparse computational graph. But all of these results make it seem likely that Jacobian sparsity is a good approximation of computational sparsity, and when combined with the fact that we have now developed efficient ways of computing them at scale, this leads us to believe that JSAEs are a highly useful approach. We would, however, still invite future work to further investigate the degree to which Jacobians, and by extension JSAEs, capture the structure we care about when analyzing LLMs.

7. Conclusion

We introduced Jacobian sparse autoencoders (JSAEs), a new approach for discovering sparse computation in LLMs in a fully unsupervised way. We found that JSAEs induce sparsity in the Jacobian matrix of the function that represents an MLP layer in the sparse basis found by JSAEs, with minimal degradation in the reconstruction quality and downstream performance of the underlying model and no degradation in the interpretability of latents. We demonstrated that the computation found by JSAEs is often highly interpretable, allowing us to see not only the concepts computed by MLPs, but also the "input concepts" which are used to compute each "output concept". We also found that Jacobian sparsity is substantially greater in pre-trained LLMs than in ran-

[2]For reference, the median value of $s_{\mathrm{x},j}$ without any interventions is 2.5.

domly initialized ones suggesting that Jacobian sparsity is indeed a proxy for learned computational structure. Lastly, we found that Jacobians are a highly accurate measure of computational sparsity due to the fact that the MLP in the JSAE basis consists mostly of linear functions relating input to output JSAE latents.

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Impact Statement

The work presented in this paper advances the field of mechanistic interpretability. Our hope is that interpretability will prove beneficial in making LLMs safer and more robust in ways ranging from better detection of model misuse to editing LLMs to remove dangerous capabilities.

Author contribution statement

Conceptualization was done by LF and LA. Derivation of an efficient way to compute the Jacobian was done by LF and LA. Implementation of the training codebase was done by LF. The experiments in Jacobian sparsity, auto-interpretability, reconstruction quality, and approximate linearity of f_s were done by LF. Qualitative examples of the computations found by JSAEs were done by TL. LA and CH provided supervision and guidance throughout the project. The text was written by LF, LA, TL, and CH. Figures were created by LF and TL with advice from LA and CH.

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A. Efficiently computing the Jacobian

A simple form for the Jacobian of the function $f_s = e_{\text{y}} \circ f \circ d_{\text{x}} \circ \tau_k$, which describes the action of an MLP layer f in the sparse input and output bases, follows from applying the chain rule. Note that here, the subscripts f_s, e_{y}, etc. denote the function in question rather than vector or matrix indices. For the GPT-2-style MLPs that we study, the components of f_s are:

1. **TopK**. This function takes sparse latents $\mathbf{s}_{\text{x}}$ and outputs sparse latents $\bar{\mathbf{s}}_{\text{x}}$. Importantly, $\mathbf{s}_{\text{x}} = \bar{\mathbf{s}}_{\text{x}}$. This step makes the backward pass of the Jacobian computation more efficient but does not affect the forward pass.

$$\bar{\mathbf{s}}_{\text{x}} = \tau_k(\mathbf{s}_{\text{x}}) \tag{6}$$

2. **Input SAE Decoder**. This function takes sparse latents $\bar{\mathbf{s}}_{\text{x}}$ and outputs dense MLP inputs $\hat{\mathbf{x}}$:

$$\hat{\mathbf{x}} = d_{\text{x}}(\bar{\mathbf{s}}_{\text{x}}) = \mathbf{W}_{\text{x}}^{\text{dec}}\bar{\mathbf{s}}_{\text{x}} + \mathbf{b}_{\text{x}}^{\text{dec}} \tag{7}$$

3. **MLP**. This function takes dense inputs $\hat{\mathbf{x}}$ and outputs dense outputs $\mathbf{y}$:

$$\mathbf{z} = \mathbf{W}_1\hat{\mathbf{x}} + \mathbf{b}_1\ , \quad \mathbf{y} = \mathbf{W}_2\phi_{\text{MLP}}(\mathbf{z}) + \mathbf{b}_2 \tag{8}$$

   where ϕ_{MLP} is the activation function of the MLP (e.g., GeLU in the case of Pythia models).

4. **Output SAE Encoder**. This function takes dense outputs $\mathbf{y}$ and outputs sparse latents $\mathbf{s}_{\text{y}}$:

$$\mathbf{s}_{\text{y}} = e_{\text{y}}(\mathbf{y}) = \tau_k\left(\mathbf{W}_{\text{y}}^{\text{enc}}\mathbf{y} + \mathbf{b}_{\text{y}}^{\text{enc}}\right) \tag{9}$$

The Jacobian $\mathbf{J}_{f_s} \in \mathbb{R}^{n_{\text{y}} \times n_{\text{x}}}$ for a single input activation vector has the following elements, in index notation:

$$J_{f_s,ij} = \frac{\partial s_{\text{y},i}}{\partial s_{\text{x},j}} = \sum_{k\ell mn} \frac{\partial s_{\text{y},i}}{\partial y_k}\frac{\partial y_k}{\partial z_\ell}\frac{\partial z_\ell}{\partial \hat{x}_m}\frac{\partial \hat{x}_m}{\partial \bar{s}_{\text{x},n}}\frac{\partial \bar{s}_{\text{x},n}}{\partial s_{\text{x},j}} \tag{10}$$

We compute each term like so:

1. **Output SAE Encoder derivative**:

$$\frac{\partial s_{\text{y},i}}{\partial y_k} = \tau_k'\left(\sum_j W_{ij}^{\text{enc}} y_j + b_{\text{enc},i}\right) W_{\text{y},ik}^{\text{enc}} = \begin{cases} W_{\text{y},ik}^{\text{enc}} & \text{if } i \in \mathcal{K}_2 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

   where $\mathcal{K}_2$ is the set of indices selected by the TopK activation function τ_k of the second (output) SAE. Importantly, the subscript k *does not* indicate the k-th element of τ_k, whereas it *does* indicate the k-th column of $W_{\text{y},ik}^{\text{enc}}$.

2. **MLP derivatives**:

$$\frac{\partial y_k}{\partial z_\ell} = W_{2,k\ell}\,\phi_{\text{MLP}}'(z_\ell)\ , \quad \frac{\partial z_\ell}{\partial \hat{x}_m} = W_{1,\ell m} \tag{12}$$

3. **Input SAE Decoder derivative**:

$$\frac{\partial \hat{x}_m}{\partial \bar{s}_{\text{x},n}} = W_{\text{x},mn}^{\text{dec}} \tag{13}$$

4. **TopK derivative**:

$$\frac{\partial \bar{s}_{\text{x},n}}{\partial s_{\text{x},j}} = \begin{cases} 1 & \text{if } j \in \mathcal{K}_1 \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

   where $\mathcal{K}_1$ is the set of indices (corresponding to SAE latents) that were selected by the TopK activation function τ_k of the first (input) SAE, which we explicitly included in the definition of f_s above.

When we combine all the terms:

$$J_{f_s,ij} = \begin{cases} \sum_{k\ell m} W^{\text{enc}}_{\text{y},ik}\, W_{2,k\ell}\, \phi'_{\text{MLP}}(z_\ell)\, W_{1,\ell m}\, W^{\text{dec}}_{\text{x},mj} & \text{if } i \in \mathcal{K}_2 \wedge j \in \mathcal{K}_1 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

Let $\mathbf{W}^{\text{enc(active)}}_{\text{y}} \in \mathbb{R}^{k \times m_{\text{y}}}$ and $\mathbf{W}^{\text{dec(active)}}_{\text{x}} \in \mathbb{R}^{m_{\text{x}} \times k}$ contain the active rows and columns, i.e., the rows and columns corresponding to the $\mathcal{K}_2$ or $\mathcal{K}_1$ indices respectively. The Jacobian then simplifies to:

$$\mathbf{J}^{\text{(active)}}_{f_s} = \underbrace{\mathbf{W}^{\text{enc(active)}}_{\text{y}} \mathbf{W}_2}_{\mathbb{R}^{k \times d_{\text{MLP}}}} \cdot \underbrace{\phi'_{\text{MLP}}(\mathbf{z})}_{\mathbb{R}^{d_{\text{MLP}} \times d_{\text{MLP}}}} \cdot \underbrace{\mathbf{W}_1 \mathbf{W}^{\text{dec(active)}}_{\text{x}}}_{\mathbb{R}^{d_{\text{MLP}} \times k}} \tag{16}$$

where d_{MLP} is the hidden size of the MLP. Note that $\mathbf{J}^{\text{(active)}}_{f_s}$ is of size $k \times k$, while the full Jacobian matrix $\mathbf{J}_{f_s}$ is of size $n_{\text{y}} \times n_{\text{x}}$. However, $\mathbf{J}^{\text{(active)}}_{f_s}$ contains all the nonzero elements of $\mathbf{J}_{f_s}$, so it is all we need to compute the loss function to train Jacobian SAEs (Section 4.1).

A.1. JSAEs with GLUs

The equations above can be easily adapted to work with gated linear units (GLUs), which are significantly more common in modern LLMs than GPT-2-style MLPs.

To do this, we modify the MLP equations like so:

$$\mathbf{g} = \mathbf{W}_{\text{g}} \hat{\mathbf{x}} + \mathbf{b}_{\text{g}} \tag{17}$$
$$\mathbf{s} = \phi_{\text{MLP}}(\mathbf{g}) \tag{18}$$
$$\mathbf{h} = \mathbf{W}_1 \hat{\mathbf{x}} + \mathbf{b}_1 \tag{19}$$
$$\mathbf{z} = \mathbf{h} \odot \mathbf{s} \tag{20}$$
$$\mathbf{y} = \mathbf{W}_2 \mathbf{z} + \mathbf{b}_2 \tag{21}$$

where $\odot$ is elementwise multiplication.

We then modify the derivatives accordingly:

$$\frac{\partial y_k}{\partial z_\ell} = W_{2,k\ell} \tag{22}$$
$$\frac{\partial z_\ell}{\partial \hat{x}_m} = h_\ell \frac{\partial s_\ell}{\partial \hat{x}_m} + s_\ell \frac{\partial h_\ell}{\partial \hat{x}_m} \tag{23}$$
$$\frac{\partial h_\ell}{\partial \hat{x}_m} = W_{1,\ell m} \tag{24}$$
$$\frac{\partial s_\ell}{\partial g_\ell} = \phi'_{\text{MLP}}(g_\ell) \tag{25}$$
$$\frac{\partial g_\ell}{\partial \hat{x}_m} = W_{\text{g},\ell m} \tag{26}$$
$$\tag{27}$$

Combining the terms again:

$$J_{f_s,ij} = \begin{cases} \sum_{k\ell m} W^{\text{enc}}_{\text{y},ik}\, W_{2,k\ell}\, (h_\ell\, \phi'_{\text{MLP}}(g_\ell)\, W_{\text{g},\ell m} + s_\ell\, W_{1,\ell m})\, W^{\text{dec}}_{\text{x},mj} & \text{if } i \in \mathcal{K}_2 \wedge j \in \mathcal{K}_1 \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

The Jacobian is then:

$$\mathbf{J}^{\text{(active)}}_{f_s} = \underbrace{\mathbf{W}^{\text{enc(active)}}_{\text{y}} \mathbf{W}_2}_{\mathbb{R}^{k \times d_{\text{MLP}}}} \cdot \underbrace{(\text{diag}\,(\mathbf{h} \odot \phi'_{\text{MLP}}(\mathbf{g}))\, \mathbf{W}_{\text{g}} + \text{diag}(\mathbf{s}) \mathbf{W}_1)}_{\mathbb{R}^{d_{\text{MLP}} \times m_{\text{x}}}} \cdot \underbrace{\mathbf{W}^{\text{dec(active)}}_{\text{x}}}_{\mathbb{R}^{m_{\text{x}} \times k}} \tag{29}$$

B. f_s is approximately linear

Consider the scalar function $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}} : \mathbb{R} \to \mathbb{R}$ which takes as input the j-th latent activation of the first SAE (i.e. $s_{\mathrm{x},j}$) and returns as output the i-th latent activation of the second SAE (i.e., $s_{\mathrm{y},i}$), while keeping the other elements of the input vector fixed at the same values as $\mathbf{s}_\mathrm{x}$. In other words, this function captures the relationship between the j-th input SAE latent and the i-th output SAE latent in the context of $\mathbf{s}_\mathrm{x}$. Geometrically, we start off at the point $\mathbf{s}_\mathrm{x}$, and we move from it through the input spaces in parallel to the j-th basis vector, and then we observe how the output of f_s projects onto the i-th basis vector. Formally,

$$f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}(x) = f_s\left(\psi(\mathbf{s}_\mathrm{x}, i, x)\right)_j \tag{30}$$

$$\psi(\mathbf{s}_\mathrm{x}, i, x)_k = \begin{cases} x & \text{if } i = k \\ s_{\mathrm{x},j} & \text{otherwise} \end{cases} \tag{31}$$

These are the functions shown in Figure 8a, of which the vast majority are linear (Figure 8b).

As we showed in Figure 8c, the absolute value of a Jacobian element nearly perfectly predicts the change we see in the output SAE latent activation value when we make a large intervention on the input SAE latent activation. However, in the same figure, there is a small cluster of approximately 2.5% of samples, where the Jacobian element is near zero, but the change observed in the downstream feature is quite large. We proceed by exploring the cause behind this phenomenon.

Note that each point in Figure 8 corresponds to a single scalar function $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ (a pair of latent indices). An expanded version of Figure 8 is presented in Figure 10. Importantly, we show the 'line', the top-left cluster, and outliers visible in Figure 8 in different colors, which we re-use in the following charts (Figures 11 and 12). It also includes 10K samples, compared to 1K in Figure 8c: as above, most samples remain on the line, but the greater number of samples makes the behavior of the top-left cluster and outliers clearer.

Figure 11 illustrates some examples of functions $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ taken from each category shown in Figure 8, i.e., the line, cluster, and outliers. The vast majority of functions belong to the line category and are typically either linear or akin to JumpReLU activation functions (which include step functions as a special case). By contrast, the minority of functions belonging to the cluster or outliers are typically also JumpReLU-like, except where the unmodified input latent activation is close to the point where the function 'jumps', so when we subtract an activation value of 1 from the input (as in Figures 8c and 10), this moves to the flat region where the output latent activation value is zero.

As we can see, the vast majority of these functions are either linear or JumpReLUs. Indeed, we verify this across the sample size of 10,000 functions and find that 88% are linear, 10% are JumpReLU (excl. linear, which is arguably a special case of JumpReLU), and only 2% are neither[3]. This result is encouraging – for a linear function, the first-order derivative is constant, so its value (i.e., the corresponding element of the Jacobian) completely expresses the relationship between the input and output values (up to a constant intercept). For the 88% of these scalar functions that are linear, the Jacobian thus accurately captures the notion of computational sparsity that interests us, rather than serving only as a proxy. And for the 10% of JumpReLUs, the Jacobians still perfectly measure the computational change we observe when changing the input latent within some subset of the input space.

While we expect the remaining 2% of scalar functions (Jacobian elements) to contribute only a small fraction of the computational structure of the underlying model, we preliminarily investigated their behavior. Figure 12 shows 12 randomly selected non-linear, non-JumpReLU $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ functions. Even though these functions are nonlinear, they are still reasonably close to being linear, i.e., their first derivative is still predictive of the change we see throughout the input space. Indeed, most of them are on the diagonal line in Figure 10.

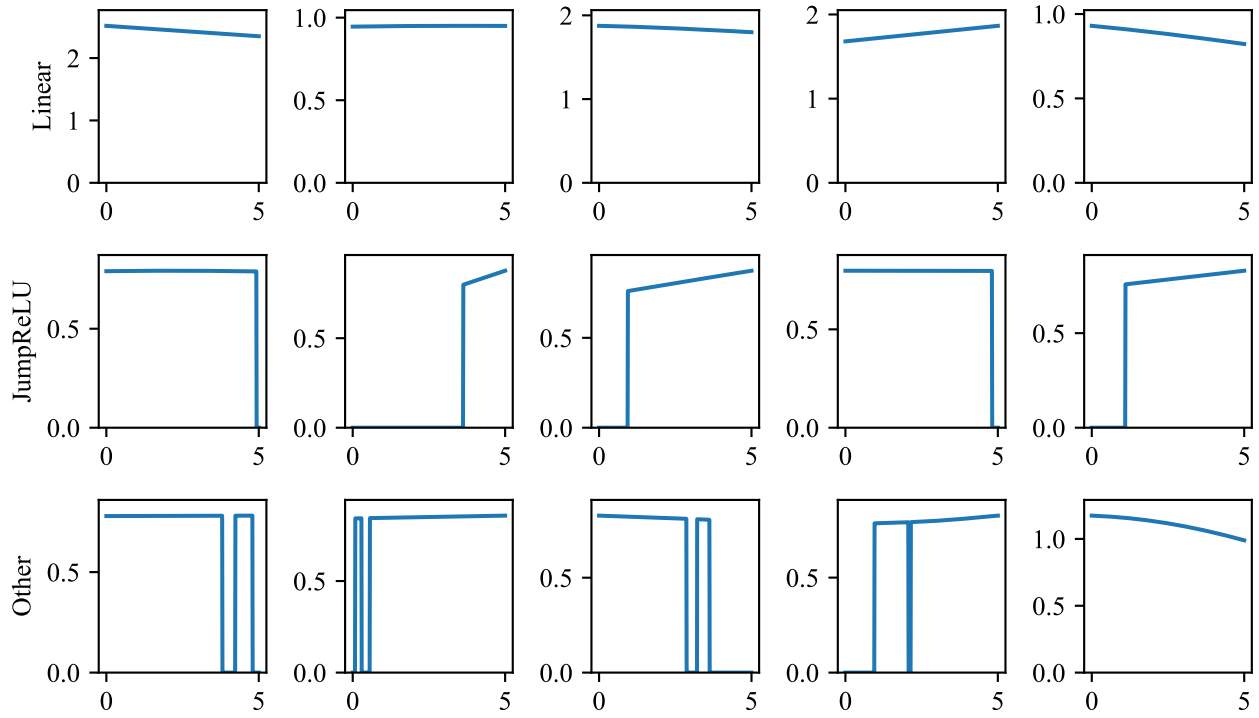


Figure 9. Additional examples of scalar functions between $s_{\mathrm{x},j}$ to $s_{\mathrm{y},i}$. The top row shows linear functions, the middle row shows JumpReLU functions, and the bottom row shows other functions. Recall that linear functions constitute a majority of the functions we observe empirically and that using JSAEs instead of traditional SAEs further increases the proportion of linear functions.

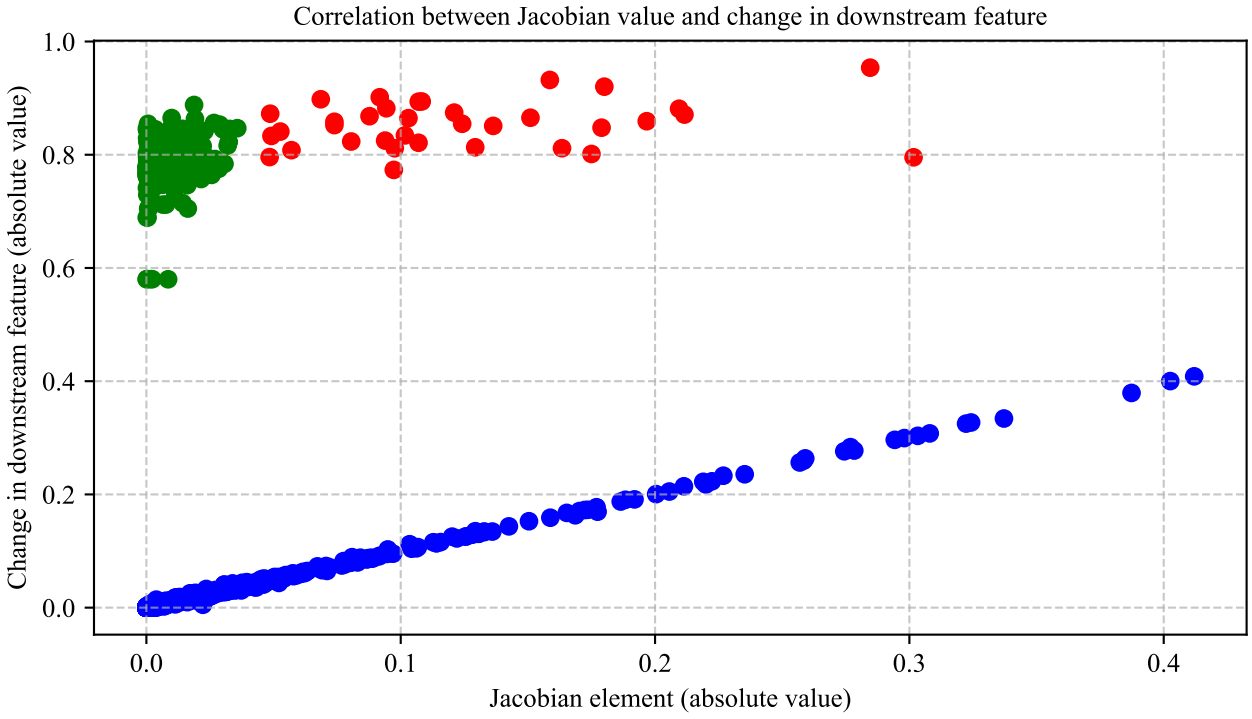


Figure 10. An expanded version of Figure 8c, measured on layer 3 of Pythia-70m. A scatter plot showing that values of Jacobian elements tend to be approximately equal to the change we see in the downstream feature when we modify the value of the upstream feature, namely when we subtract 1 from it. Each dot corresponds to an (input SAE latent, output SAE latent) pair. Unlike Figure 8c, this figure colors in the dots depending on which cluster they belong to – blue for "on the line", green for "in the cluster", red for "outlier". Additionally, this figure contains 10,000 samples (rather than 1,000 as in Figure 8c), which allows us to see more of the outliers and edge cases, though at the cost of visually obfuscating the fact that 97.5% of the samples are on the diagonal line, 2.1% are in the cluster, and 0.4% are outliers.

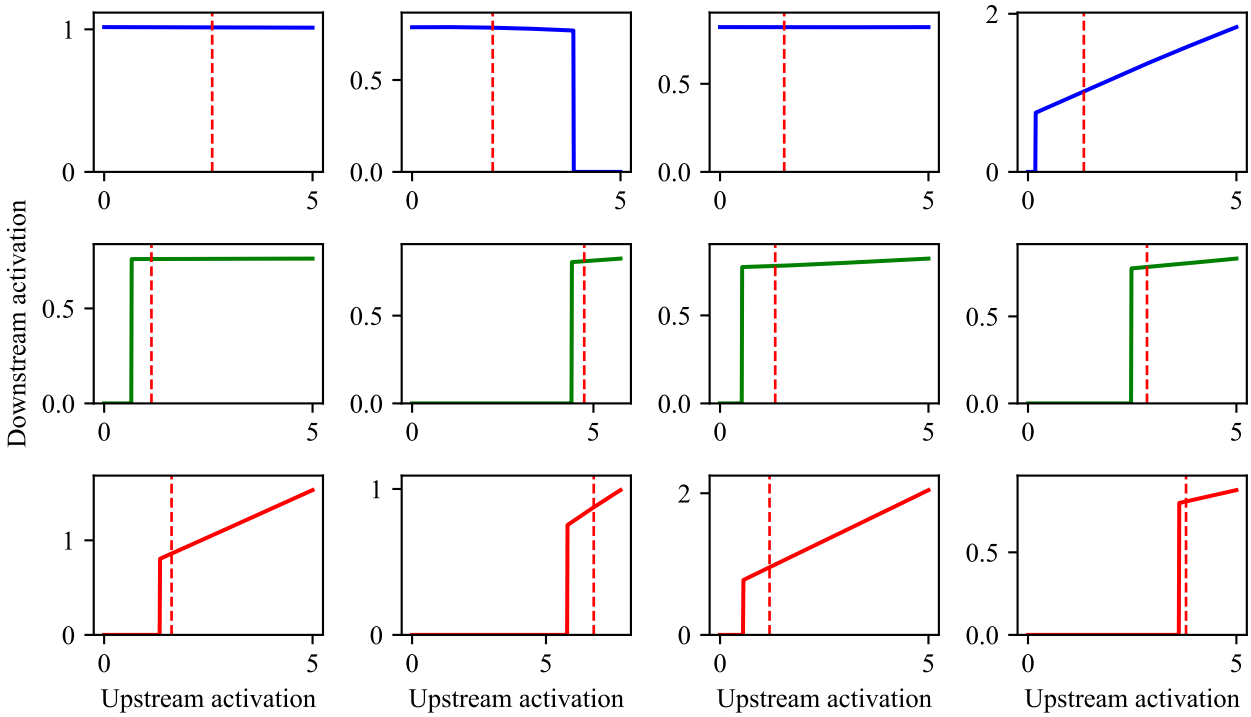


Figure 11. A handful of $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ functions corresponding to the points in Figure 10. The color matches the group (and therefore the color) they were assigned in Figure 10. The red dashed vertical line denotes $\mathbf{s}_{\mathrm{x},i}^{(l)}$, i.e. the activation value of the SAE latent before we intervened on it. Note that the functions are not selected randomly but rather hand-selected to demonstrate the range of functions. We will quantitatively explore what proportion of $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ functions have which structure in other figures.

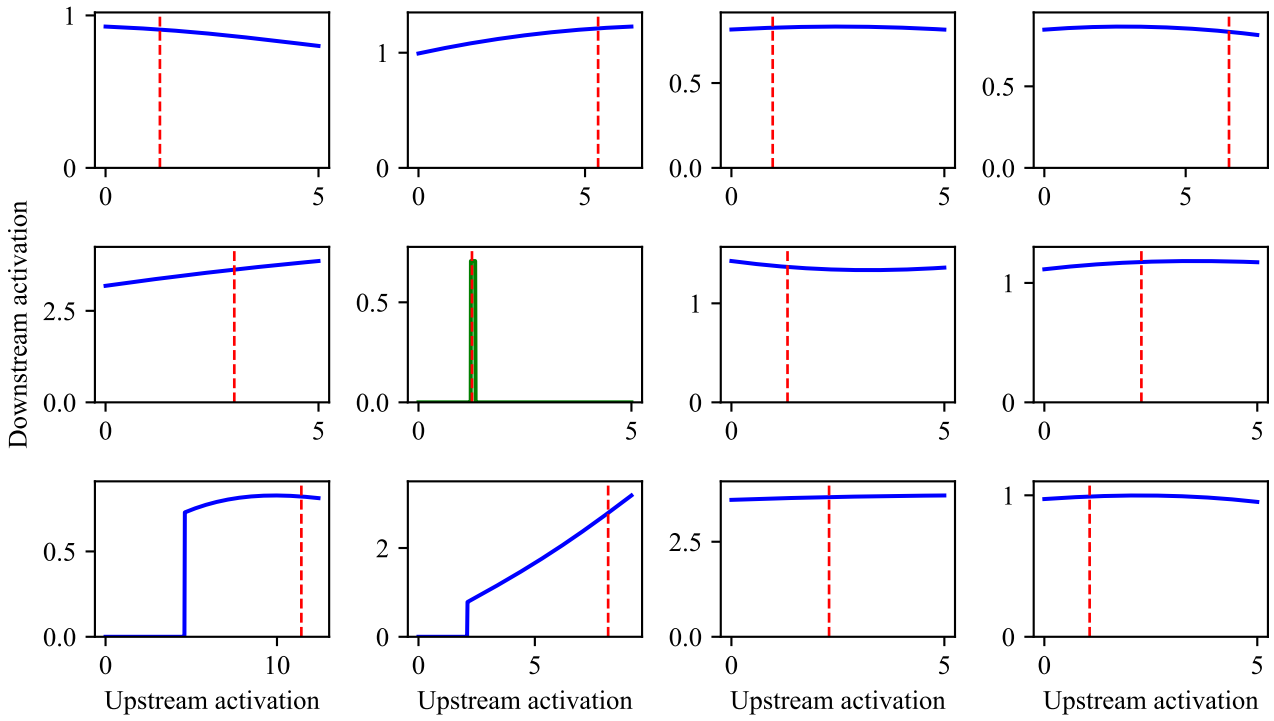


Figure 12. A random selection of the non-linear, non-JumpReLU $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ functions. Note that non-linear, non-JumpReLU functions only constitute about 2% of $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$ functions. Even though these functions are clearly somewhat non-linear, their slope does still change quite slowly for the most part, which means that a first-order derivative at any point in the function is still reasonably predictive of the function's behavior in at least some portion of the input space (though there are some rare exceptions). The color again matches the group (and therefore the color) they were assigned in Figure 10; the red dashed vertical line denotes $\mathbf{s}^{(l)}_{\mathrm{x},i}$, i.e. the activation value of the SAE latent before we intervened on it.

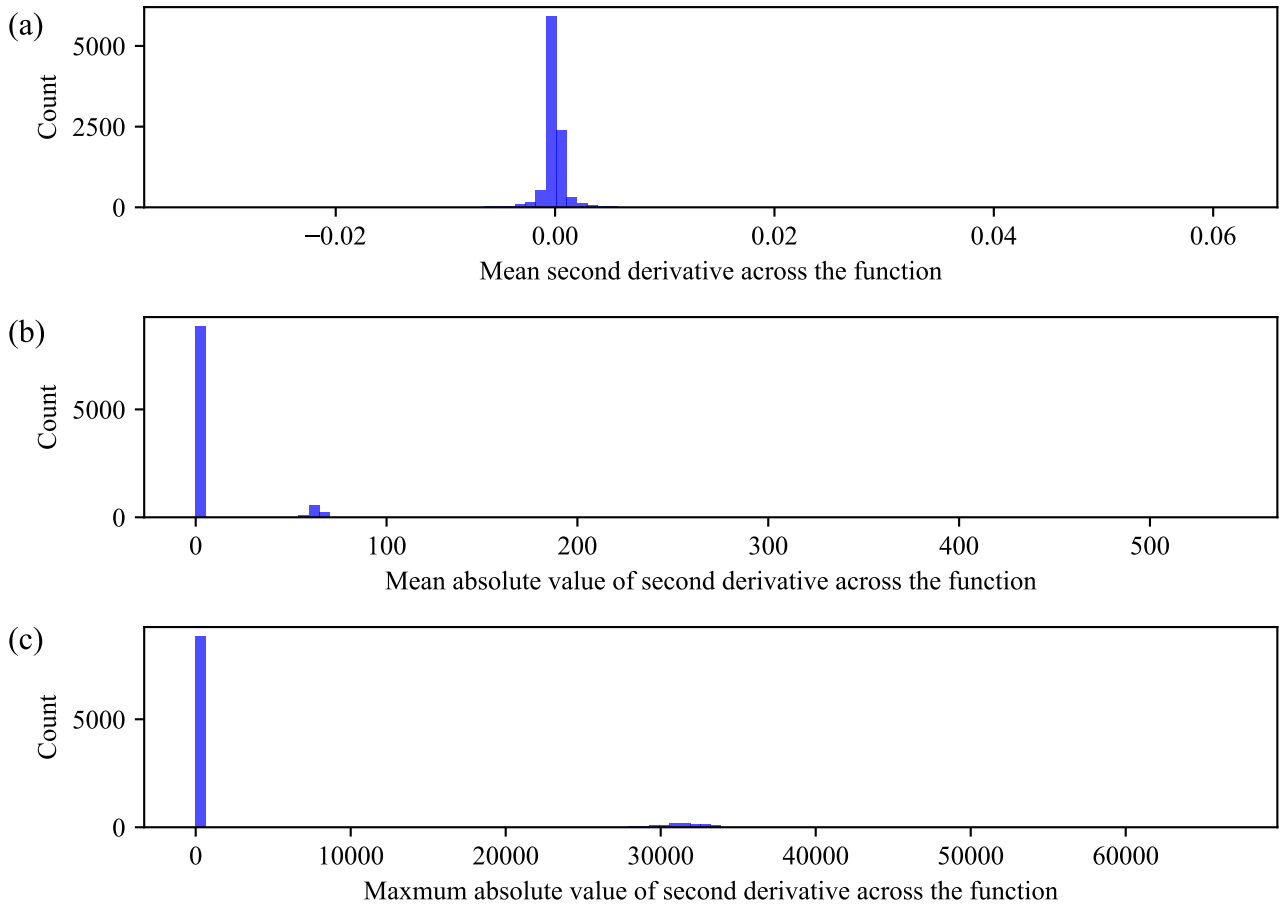


Figure 13. Distribution of second-order derivatives of functions $f_{s,(i,j)}|_{\mathbf{s}_x}$. Includes all functions, regardless of whether they are linear, JumpReLU, or neither. For a version that only includes non-linear, non-JumpReLU functions, see Figure 14. (a) The mean of the second-order derivative over the region of the input space. (b) The mean of the absolute value of the second-order derivative over the region of the input space. (c) The maximum value the second-order derivative takes in the region of the input space. Note that we are approximating the second derivative by looking at changes over a very small region (specifically 0.005), i.e., we do not take the limit as the size of this small region goes to zero; this is important because derivatives which would otherwise be undefined or infinite become finite with this approximation and therefore can be shown on the histograms. Also, we note that the means and maxima are taken over the region of the input space in which SAE features exist; see the footnote on page 15.

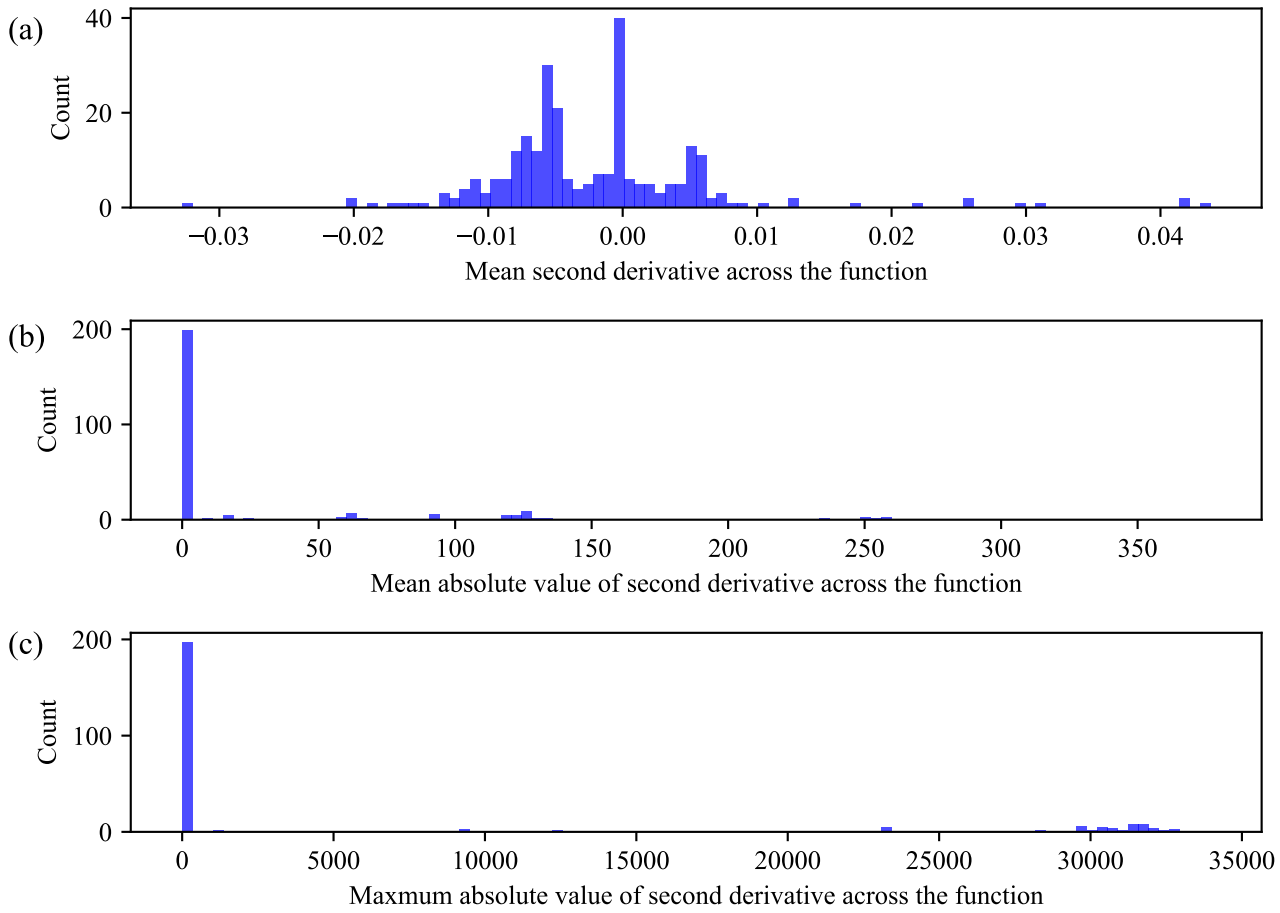


Figure 14. Distribution of second-order derivatives of functions $f_{s,(i,j)}|_{\mathbf{s}_\mathbf{x}}$. Unlike Figure 13, this figure only includes the subset of the functions that are neither linear nor JumpReLU=like. (a) The mean of the second-order derivative over the region of the input space. (b) The mean of the absolute value of the second-order derivative over the region of the input space. (c) The maximum value the second-order derivative takes in the region of the input space. Note that we are approximating the second derivative by looking at changes over a very small region (specifically 0.005), i.e. we do not take the limit as the size of this small region goes to zero; this is important because derivatives which would otherwise be undefined or infinite become finite with this approximation and therefore can be shown on the histograms. Also, we note that the means and maxima are taken over the region of the input space in which SAE features exist; see the footnote on page 15.

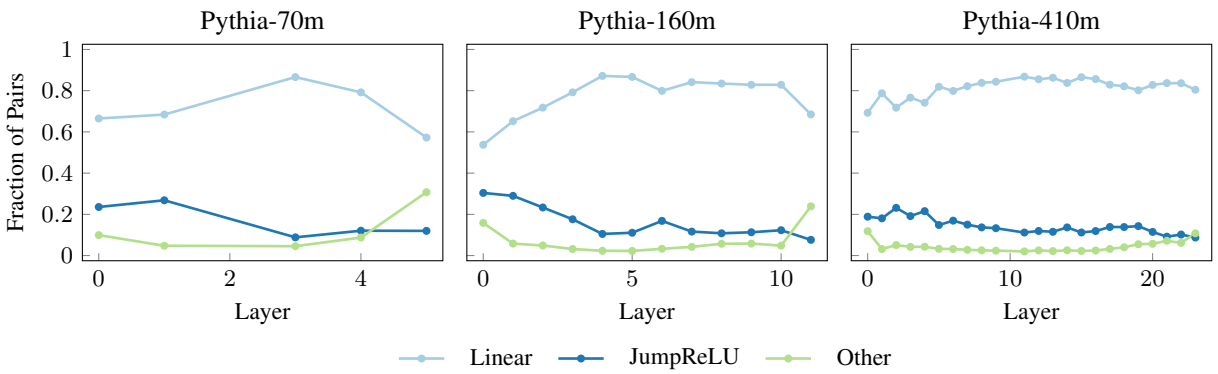


Figure 15. The fractions of Jacobian elements that exhibit a linear relationship between the input and output SAE latent activations, a JumpReLU-like relationship, and an uncategorized relationship, as described in Section 5.4. Here, we consider Jacobian SAEs trained on the feed-forward network at different layers of Pythia-70m, 160m, and 410m with fixed expansion factors $R = 64$ and $k = 32$. We computed the fractions over 1 million samples.

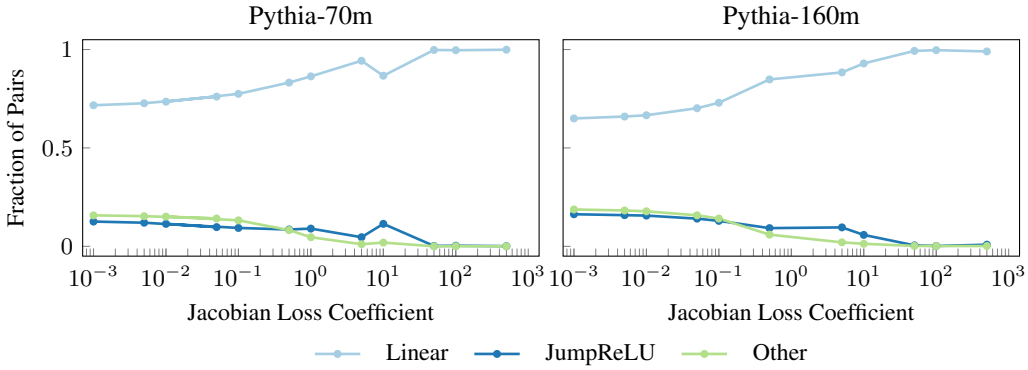


Figure 16. The fractions of Jacobian elements that exhibit a linear relationship between the input and output SAE latent activations, a JumpReLU-like relationship, and an uncategorized relationship, as described in Section 5.4. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m (left) and layer 7 of Pythia-160m (right), with fixed expansion factors $R = 64$ and $k = 32$ and varying Jacobian loss coefficient (Section 4). We computed the fractions over 1 million samples.

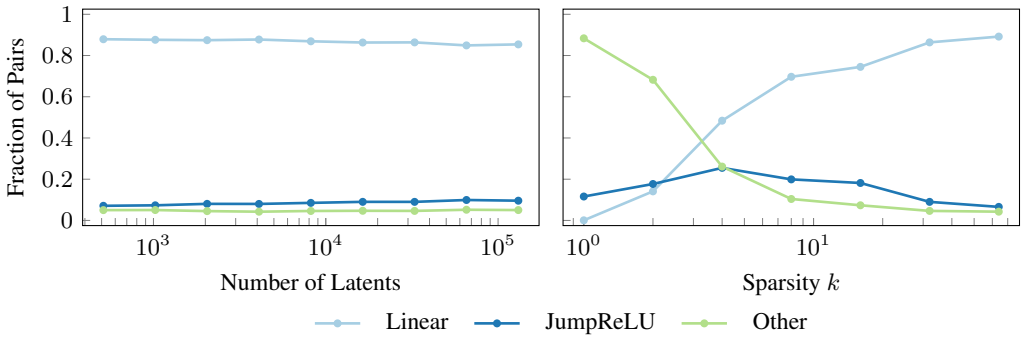


Figure 17. The fractions of Jacobian elements that exhibit a linear relationship between the input and output SAE latent activations, a JumpReLU-like relationship, and an uncategorized relationship, as described in Section 5.4. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m with varying expansion factors (and hence numbers of latents; left) but fixed sparsities $k = 32$, and varying sparsities but fixed expansion factors $R = 64$ (Section 4). We computed the fractions over 1 million samples.

We can measure this more precisely by looking at the second-order derivative of $f_{s,(i,j)}|_{\mathbf{s}_\mathrm{x}}$. A zero second-order derivative across the whole domain would imply a linear function and, therefore, perfect predictive power of the Jacobian, while the larger the absolute value of the second-order derivative, the less predictive the Jacobian will be. This distribution is shown in Figure 13. The same distribution, which only includes the non-linear, non-JumpReLU functions, is shown in Figure 14. On average, the second derivative is extremely small for all features and effectively zero for the vast majority.

C. Qualitative examples of the computations discovered by JSAEs

A common approach to interpreting LLM components like neurons or SAE latents is to collect token sequences and the corresponding activations over a text dataset (e.g., Yun et al., 2021; Bills et al., 2023). For example, the greatest latent activations may be retained, or activations from different quantiles of the distribution over the dataset (Bricken et al., 2023; Choi et al., 2024; Paulo et al., 2024).

We determined the set of 'top' output SAE latent indices by collecting the mean absolute values of non-zero Jacobian elements over a text dataset and sorting the output latents in descending order. Then, for each output latent, we found the input SAE latents that were most strongly connected to the output latent, again by sorting the input latents in descending order of the mean absolute value of non-zero Jacobian elements over the dataset. Finally, for both the output and input latents, we collected the individual latent activations over text samples with a context length of 16 tokens, retaining samples where at least one token produced a non-zero activation for the SAE latent. We chose a short context length to conveniently display the examples in a table format, and display here the top eight examples for each latent index, sorting the examples in descending order of the maximum latent activation over its tokens.

Each of the following figures comprises a table for a single output SAE latent (in pink), and a series of tables for the input latents with the greatest influences on the output latent, as determined by the mean absolute value of non-zero Jacobian elements. Conceptually, one may consider each figure as describing a single 'function', where the output and input latents represent the function output and inputs, respectively. Each table within the figure of examples displays a list of at most 12 examples, each comprising 16 tokens; we exclude the end-of-sentence token for brevity. The values of non-zero Jacobian elements and the activations of the corresponding input and output SAE latent indices are indicated by the opacity of the background color for each token. We take the opacity to be the element or activation divided by the maximum value over the dataset, i.e., all the examples with a non-zero Jacobian element for a given pair of input and output SAE latent indices. For clarity, we report the maximum element or activation alongside the colored tokens.

[3]Note that we are testing whether functions are linear or JumpReLUs only in the region of input space within which SAE activations exist. In particular, this means that we are excluding negative numbers. More specifically, the domain within which we test the function's structure is $[0, \max(5, \mathbf{s}^{(l)}_{\mathrm{x},i} + 1)]$. In 92% of cases, $\mathbf{s}^{(l)}_{\mathrm{x},i} + 1 < 5$; the median $\mathbf{s}^{(l)}_{\mathrm{x},i}$ is 2.5.

Example tokens	Max. activation
W enn Sie auf We iter k lic ken ist I hr Bon us ver	5.596
ro A14ber fl A14ss ig gew orden ist . Lau f zeit 36 Mon ate	5.542
es muss von einer aus lA nd ischen Bank sein . Log ischer we ise	5.522
. Sie kAnnen sich j eder zeit ab mel den . i . Click Here	5.387
bei der Be re it st ell ung unse rer D ien ste . \n	5.315
s icher lich sch on ge hA rt haben , kann es sehr ze	5.275
G eld aus . \n J ump No multiple accounts or free bonuses in a	5.254
we il be borrowing food from China , next .. Oh what a web we	5.247
k auf en will , kann ich tro tz dem das Gold V isa be	5.225
bef A14r wort ete auch die vol l stA nd ige K ontrol le A14ber	5.223
bonus codes 2019 , so dass ein We g in ein W ett b A14	5.208
re Imm ob il ie in Sp an ien auf ih ren N amen beh	5.207

(a) The top 12 examples that produce the maximum latent activations for the output SAE latent with index 34455.

Example tokens	Max. activation
dam it ein ver stand en , dass wir Cookies ver w enden . Die	1.292×10^{1}
we isen , dass Sie nicht mehr als 11 Mon ate a uss er hal	1.229×10^{1}
st super auf me ine W A14n sche e ing eg ang en . Ins	1.173×10^{1}
Golden V isa), die gle ich zeit ig er la ub t , dass	1.167×10^{1}
f A14r das Gold V isa in 2013 er lass en . Zu dies em	1.152×10^{1}
W enn Sie auf We iter k lic ken ist I hr Bon us ver	1.139×10^{1}
age : W enn ich die Imm ob il ie durch eine Ges ells chaft	1.134×10^{1}
ung abs ch icken d auer b ren ner zur A14ck le hn en :	1.132×10^{1}
lich die ID K arte ver l Ang ert werden , das sehr s chn	1.126×10^{1}
te " \n " A uf alle W A14n sche e ing eg ang en	1.125×10^{1}
keine oder an le icht er f A14 ll bare B eding ungen g ek	1.124×10^{1}
hat Sp an ien ein Ges etz er lass en , dass einem N icht	1.116×10^{1}

(b) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 39503. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 15130 tokens, and has a mean of 3.966×10^{-1} (rank 0 for the output SAE latent) and a standard deviation of 7.743×10^{-2} over its non-zero values.

Example tokens	Max. activation
ZA14 rich St utt gart Le ipzig / H alle	6.834
OTC cad ets , students , and staff of Ans bach Middle High School	6.788
Out reach announced that two groups of students from Ans bach Middle High School won	6.693
ride from Zur ich to Rh ine Falls (with a small section of our	6.679
aud . U55 Brand en bur ger Tor Bus 20000 U hr die	6.644
at the Ans bach Middle High School track . \n G ram my award winning	6.566
, Ans bach , Germany , on May 18 , 2015 . Fry e was	6.546
parents to attend . \n W ies b aden students find math a little fish	6.480
into the Gi essen b acht al valley . On the trail along the edge	6.471
was the first case for the b MC Kob len z office . Capt .	6.448
ol ie . Follow the signs Brand en bur ger Tor Brand enburg Gate or	6.423
for the Kob len z office . \n b MC ' s Captain Dennis Brand	6.391

(c) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 3387. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 10355 tokens, and has a mean of 3.437×10^{-1} (rank 1 for the output SAE latent) and a standard deviation of 2.873×10^{-2} over its non-zero values.

Example tokens	Max. activation
, Dr Alex German and his team found the main reason for that is the	1.149×10^{1}
for Nuclear Physics , R up recht Kar ls University , the German Cancer Research	1.142×10^{1}
, Chris Kend rick , Sally Anne F itter and Penny German . \n More	1.137×10^{1}
1 . 2 in Czech oslov akia , 1 . 1 in the German Democratic	1.119×10^{1}
mind , intellect "), from Pro to– German ic ∗ mund iz , ∗	1.089×10^{1}
and Hyde F li ppo ' s Aut ob ahn articles from The German Way and	1.084×10^{1}
" L ili Mar lene ", the song that German and American soldiers both loved	1.075×10^{1}
ls– Univers itAt , the German Cancer Research Center (DK FZ),	1.070×10^{1}
. In addition , Dr German ' s team also confirmed that cats ate about	1.068×10^{1}
Art . 3 of the German Constitution that addresses equality before the law . \n	1.064×10^{1}
under constant international external laboratory control of the German acc reditation system In stand .	1.052×10^{1}
But the history of carn ival can be traced back to German ic tribes who	1.051×10^{1}

(d) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 41811. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 764 tokens, and has a mean of 1.619×10^{-1} (rank 2 for the output SAE latent) and a standard deviation of 8.654×10^{-3} over its non-zero values.

Example tokens	Max. activation
National Archives report " Hit ler ' s Shadow : Nazi War Crim inals ,	1.203×10^{1}
Th ousands of Holocaust victims transported to Nazi concentration camps by a French railway company	1.195×10^{1}
Air Corps after WW II (where he survived a year as a Nazi prisoner	1.170×10^{1}
a name inspired by the chief Ukrainian Nazi leader , Ste pan Band era)	1.168×10^{1}
Powers refused cooperation with the company until connections with Nazi Germany were severed . \n	1.155×10^{1}
orrow , the World ! ," from 1944 , about a teen – age Nazi	1.145×10^{1}
, 000 French Jews to Nazi concentration camps , though experts disagree on its degree	1.145×10^{1}
of the Jewish people during the Nazi era it was the work of white Europeans	1.118×10^{1}
computing pioneer , and his development of a system to crack Nazi codes in order	1.099×10^{1}
it had no effective control over operations during the Nazi occupation from 1940 to 1944	1.090×10^{1}
it with heavy water + Nazi chemicals , prayed to god for assistance , struck	1.079×10^{1}
symbol of Russian resistance to Nazi invasion . Russia ' s " Window on the West	1.067×10^{1}

(e) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 32619. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 316 tokens, and has a mean of 1.518×10^{-1} (rank 3 for the output SAE latent) and a standard deviation of 1.193×10^{-2} over its non-zero values.

Example tokens	Max. activation
owned by the von W aken itz family in 14 34 . The same family	1.605×10^{1}
World Nuclear Industry Status Report 2013 , July . \n 8 David von Hipp el	1.526×10^{1}
as Mart a , one of the singing von Tra pp children . She i 12	1.509×10^{1}
knockdown kiss is administered when Von locks lips with GG way more enthusi astically than	1.437×10^{1}
, I assume . Gre ta Gar bo is here with Von , and not	1.427×10^{1}
HATE THAT I DON ' T HATE YOU " \n Fel ix von	1.423×10^{1}
was already published 150 years ago by the german physician Herm ann von Hel	1.388×10^{1}
new life by Edward von L A ng us and augmented reality technologies , exploring	1.377×10^{1}
(19 28 , Joseph von Stern berg ; silent). A T ribute to	1.366×10^{1}
ily Von has again been sight ed in that infamous suit . Was Metro econom	1.352×10^{1}
artists and performers , known as the Castle von Tra pp , will im min	1.341×10^{1}
ung . D egen feld – F ests chr ift , Vienna : von Lag	1.320×10^{1}

(f) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 63157. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 84 tokens, and has a mean of 1.479×10^{-1} (rank 4 for the output SAE latent) and a standard deviation of 8.717×10^{-3} over its non-zero values.

Example tokens	Max. activation
Dum ont aG John Ax el rod , MS NBC al um now at Berlin	1.040×10^{1}
for determining adhesive and tensile strengths (w inner of the Berlin– Br anden	1.026×10^{1}
That Dem on Within was selected for the 64 th Berlin International Film Festival Pan	1.025×10^{1}
M irt l , and M . Sch mid , eds . Berlin : Springer	1.017×10^{1}
processes in tropical forests (pp . 153– 172). Springer , Berlin ,	1.011×10^{1}
Twenty– five years after the fall of the Berlin wall in 1989 , Cont	1.008×10^{1}
the Berlin Wall : legacy SA– 2 , SA– 3 , SA–	9.963
emerged after the collapse of the Berlin Wall and the end of the cold war	9.932
fall of the Berlin Wall was a great spur to Germany , though it took	9.898
exhibitions in museums and galleries in Cape Town , Johannes burg , Berlin and C	9.885
of the following options for the Berlin to Sch ild ow route : Michel in	9.885
t have to get a new one (with the Berlin citizen center , it	9.847

(g) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 63657. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 195 tokens, and has a mean of 1.387×10^{-1} (rank 5 for the output SAE latent) and a standard deviation of 6.091×10^{-3} over its non-zero values.

Example tokens	Max. activation
no zzles , are known from the Mercedes G LA . In addition , Mercedes	3.361
an open end– hole . \n Moreover , when cathe ters of this type	2.830
N asm ith . machine rollers for conveyor belt The invention relates , inter alia	2.829
cathe ters which have a closed distal end . The principles of the invention therefore	2.757
are also suitable . \n If it is possible for you to get your blood	2.487
analog matching devices can be used advantage ously for and monitoring and diagnostics of process	2.399
be the case that such support can be secured , for example , in cases	2.249
ic acid . Pro per ventilation and her met ically sealed production apparatus are therefore	2.240
of residence , in particular copyright , data protection and competition law . The provider	2.234
products of this type work great for increasing your store ' s revenue , as they	2.133
action by the customer or from errors in the information provided by the latter .	2.113
optionally a rear axle differential lock . \n In contrast to the prospective main competitor	2.099

(h) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 7969. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 331 tokens, and has a mean of 1.322×10^{-1} (rank 6 for the output SAE latent) and a standard deviation of 1.176×10^{-2} over its non-zero values.

Example tokens	Max. activation
and a private one . With a public key , it A~ s possible to	5.947
Cons olid ation 2592 . In the last years of its active life ,	5.068
forward the realization of artistic works linked to this whole context , taking into account	4.883
test different runtime environments . As a consequence , cloud– based tool ing currently	4.585
release I recommend to have a look at J SF– Spring (http ://	4.558
customer satisfaction . \n What is more , with the app , it ' s possible	4.501
of the hotel . On the hotel land it is possible to camp in tents	4.438
introduce additional modifications do not forget about re– publ ishing the whole site so	4.426
change is requested , this tool enables to know which other requirements , design and	4.418
structures / steel constructions , according to the highest requirements . \n Dep ending on	4.370
2O3 , or , more precisely , a mixture of NO and NO	4.309
if you have any questions (you can also have a look at our website	4.272

(i) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 18964. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 3502 tokens, and has a mean of 1.209×10^{-1} (rank 7 for the output SAE latent) and a standard deviation of 2.073×10^{-2} over its non-zero values.

Example tokens	Max. activation
will enjoy music inspired in or by America , France , Austria , Scotland ,	1.170×10^{1}
in the world since its adoption by the Austrian army in 1977 . Now available	1.116×10^{1}
, Australia , China , Austria , England , Israel and Ireland . \n Through	1.112×10^{1}
teaching the basics of K ung Fu in Russia , Austria , Spain , China	1.053×10^{1}
occupied areas in 1848 ; by the 1880 s , the Austrian– govern ed	1.042×10^{1}
Russia , and American Ambassador to Austria , William Each o , we had the	1.033×10^{1}
, which traces its roots to the work of the Cold War Austrian– American	1.016×10^{1}
released in Germany , Austria and Switzerland . \n Irish man in America was recorded	1.014×10^{1}
W ahl led to an invitation by the Austrian Ministry of Culture , for myself	9.958
of State from Austria , Poland , and Hungary , Min isters from Israel and	9.951
practice first abroad , in countries such as Spain , Germany or Austria , where	9.771
. \n The Che chen youth , who came to Austria as a refugee ,	9.769

(j) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 28112. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 200 tokens, and has a mean of 1.156×10^{-1} (rank 8 for the output SAE latent) and a standard deviation of 8.429×10^{-3} over its non-zero values.

Example tokens	Max. activation
Reynolds & Jake Gy ll en ha al Ar rives ! \n Earlier tonight during	9.162
Gy ll en ha al , Ryan Reynolds , and Rebecca Ferguson star in the	8.938
Jim Mc El re ath , Mario And ret ti , Gary Bet ten haus	8.771
M . ; Sp icken he uer , A . ; W agen f A14	8.518
followed by Nick H ohl be in who finished fourth . \n If Friday night	8.185
aud . U 55 Brand en bur ger Tor Bus 200 00 U hr die	8.145
the ritual He iden j acht en are not really a closed chapter of history	8.071
the husband to the late Mary Jane Hol tz cl aw . Sc oop was	8.004
aG Chris Har low ... Eric F ink be iner ... Mike Deutsch , FAA	7.912
ding Music Pro ducer and Artist LA EL , Jeff Sch nee we is ,	7.688
team is only hitting . 242 as a team . Bob Stein b ock joined	7.678
Red d , Tina Den ise L oll is , Timothy Ray Hol tz cl	7.668

(k) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 4287. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 1820 tokens, and has a mean of 1.131×10^{-1} (rank 9 for the output SAE latent) and a standard deviation of 2.500×10^{-2} over its non-zero values.

Example tokens	Max. activation
Mah avir Mer i Dr is hti Me in . One afternoon he was resting	1.030×10^{1}
haben . \n Un se rer Me in ung nach wird es in den nA	9.290
waukee Magazine prof iled Le in bach and the UEC last year . \n	8.524
about Milwaukee ' s Urban Ec ology Center ? \n Ken Le in bach ,	7.583
DO KO PY AR BH ARE AL FA ZO me in bol new	7.505
and song writing team of P ia Le in onen and Jon i T ial	7.337
on . April 27 , 2015 . By . Dom Ein horn . Cos m	6.740
term growth and sustainability . Luc as And S hel by . Ein Smith .	6.319
– top casino sites worldwide Das V ere in ig te K An ig reich	6.226
z ond heid / vo eding – met – we in ig – cal orie	6.008
inated A . C . Ke in m ies as their candidate . \n ment	5.644
my heart to live . \n D ilon me in tum ap ni bet abi	5.064

(l) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 62769. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 48 tokens, and has a mean of 1.121×10^{-1} (rank 10 for the output SAE latent) and a standard deviation of 7.225×10^{-3} over its non-zero values.

Example tokens	Max. activation
op316(1867). ViennaPO/CarlosKleiber.\n	1.541×10^{1}
,JapeK rystalKlear,KormacâsBig	1.460×10^{1}
,JordanKleeman, goes over some of the upcoming features coming to the	1.445×10^{1}
GoldenWings Music. Following his early beginnings onE el keKlijnâ	1.431×10^{1}
apenem resistant Pseudomonas aeruginosa (CRPA),Klebs iella pneumoniae(	1.373×10^{1}
drink. I bring myKleanK ante en with me everywhere and use	1.336×10^{1}
inaG arr ig ues); 2009– Barcelona(w/KlepacÌ	1.293×10^{1}
jin(w/KlepacÌ); 2010– Budapest(w/Med	1.263×10^{1}
.\nDogpoisonNo.2: In sect icides.\nFle	5.116
there and no one was boarding yet. We parked the car, schle	4.971
mark the perm al ink.\nFle as or even worse, we	4.956
left.\nGle be Rd South to Route 1. Turn right onto	4.754

(m) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 14871. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 30 tokens, and has a mean of 1.095×10^{-1} (rank 11 for the output SAE latent) and a standard deviation of 9.772×10^{-3} over its non-zero values.

Example tokens	Max. activation
LuciaRodriguez(Comcast)asChair– Elect; EmmaPfister(	1.913×10^{1}
another clever design fromPfister, but it's really hard to set	1.897×10^{1}
cast Administration fromBoston University.\nEmmaPfister(Treasurer	1.854×10^{1}
tips/my college application essay one of my jobs atPfizer since	1.817×10^{1}
,Pfizer, andCompumedics Limited, among others.\n Profile	1.781×10^{1}
.Pfannkuch andM.O.J.Thomas, eds	1.711×10^{1}
case inPfleiderer andIVGImmobilien. The	1.685×10^{1}
amore; RonaldBrautigam, pf; LeovanDoes	7.684
PenelopeThwaites, pf.\nMozart, W	6.898
).YujaWang, pf.\nRachmaninov,	6.714
ino, pf.\nRespighi, O.Belkis	6.594
Previn, pf.\nBoccherini, L. String	6.460

(n) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 32693. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 24 tokens, and has a mean of 1.092×10^{-1} (rank 12 for the output SAE latent) and a standard deviation of 9.177×10^{-3} over its non-zero values.

Example tokens	Max. activation
A . A . planned , coordinated and schm oo zed for four months	2.058×10^1
there and no one was boarding yet . We parked the car , sch le	2.045×10^1
intervention . For most of us poor schm ucks it ' s just something	2.004×10^1
– to– speech . Now print out a copy of this page , sch	1.945×10^1
st inky melting stage where you can just schm ear it on crust y	1.934×10^1
Paul VI for he res y , schism and scandal ten years ago ,	1.926×10^1
slate , sch ist , quartz ite , and limestone on the west ; met	1.909×10^1
he res y , schism and scandal , it is in fact more grave	1.893×10^1
is needed to schirm ish on a 3 area front . USSR should in	1.881×10^1
as what the French call changes to the "schol astic rhythms ." \n	1.877×10^1
' s sch tick was new and the moment was right ," and that we	1.876×10^1
ac ris , ro bin schul z , will . i . am ,	1.849×10^1

(o) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 30568. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 449 tokens, and has a mean of 1.049×10^{-1} (rank 13 for the output SAE latent) and a standard deviation of 1.109×10^{-2} over its non-zero values.

Example tokens	Max. activation
31 . \n Ra ich J W , Sch les inger W H , 1992 .	1.313×10^1
Health and Care , the Sch les wig– Hol stein Ministry for Social Affairs	1.276×10^1
Productions celebrated Justin Sch leg el ' s birthday by presenting him with a very special	1.274×10^1
from Politics and Society to Culture and Entertainment . Ber lin : Sch les inger	1.262×10^1
irling and mixing and clear mixing lines visible in it (Sch lie ren as	1.247×10^1
K yle Sch war ber may be out for the season , but the home	1.231×10^1
a Sab ot age . Arnold Sch war zen eg ger is back and looking	1.229×10^1
21) but Sch les inger ' s pla te number suggests a earlier publication date	1.224×10^1
the ' one field ' target . \n Just about one field by Sch war	1.222×10^1
ville , Texas , Sch re iner University is a small four year private college	1.218×10^1
) spoke at an event of B DO 10 . Sch if ff ah	1.211×10^1
, Mike Sch re iber and countless more . She volunteers at D ade Correction	1.203×10^1

(p) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 47756. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 451 tokens, and has a mean of 1.040×10^{-1} (rank 14 for the output SAE latent) and a standard deviation of 1.167×10^{-2} over its non-zero values.

Example tokens	Max. activation
li L ai a~K34 al~ aIJi e~ i aIJI a12l aIJI c12 al	1.484×10^{1}
e~N , a a ei ah1 eo aL aI cI e~	1.478×10^{1}
L ij ao a34I a1 l j aI e34 34 . e~	1.422×10^{1}
a N cl a i al a14 eGl a~~1~ to be , ei cK~a	1.419×10^{1}
aHa IeL cl e~ aa1 a~e~~ IIJ : I suspect	1.385×10^{1}
: content cK~ a12l aIJi e~ i , e~ co cL a12 k ,	1.385×10^{1}
l~e~e~N a12 a1 e~ i , iG a a G eI	1.379×10^{1}
Ii aoi k). \n aL I a aL l Ig eg eLa	1.378×10^{1}
co aI a1 L , ei e~ co I12 e a1 i	1.374×10^{1}
aI aL a I e ao I LH i14I 2J li eg12	1.374×10^{1}
the contents page (cL a12 k e). a a , e~	1.368×10^{1}
aIJi e~ i aii a12l al e~ Na1 1~ aHN , aH :	1.362×10^{1}

(q) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 59459. Paired with the output SAE latent with index 34455, the Jacobian element is non-zero for 1602 tokens, and has a mean of 1.036×10^{-1} (rank 15 for the output SAE latent) and a standard deviation of 1.771×10^{-2} over its non-zero values.

Figure 18. The top 12 examples that produce the maximum latent activations for the output SAE latent with index 34455, and the input SAE latents with which the mean values of the corresponding Jacobian elements are greatest. The Jacobian SAE pair was trained on layer 15 of Pythia-410m with an expansion factor of $R = 64$ and sparsity $k = 32$. The examples were collected over the first 10K records of the English subset of the C4 text dataset with a context length of 16 tokens.

Example tokens	Max. activation
kevAl retro fArgadefArdenkAnslige . Barn	4.636
v lig ut s ikt AverLipnos j An . Mys ig och barn	4.549
och fArfAr st Ark are , upp till 20timmar ut an	4.370
kar , men du mAste vara in log g ad som an st Al ld	4.337
Det kan fin nas fl er pro jek t dArLars Bank vall med ver	4.165
C ancel . E tt ex emp el pA fore x hand el Ar att	4.149
Spes ial sp ill ut val gav m ors om me og	4.121
atal ogen under Downloads i men yn dArdu en k elt bl A	4.080
A till A ! Cong rat z Eva Ny strAm and Ad riel Young !	4.037
kA pa Euro och s Al ja US Dollar . \n Results per trade binary	3.989
v An lig at mos f Ar och sk A na rum , dock kans	3.985
i smak pA det som ble ig jen . \n E arn ed the	3.977

(a) The top 12 examples that produce the maximum latent activations for the output SAE latent with index 64386.

Example tokens	Max. activation
Spes ial sp ill ut val gav m ors om me og	1.218×10^{1}
ass el og and en dans ke kon ge af den Gl252 ; cks	1.175×10^{1}
i smak pA det som ble ig jen . \n E arn ed the	1.137×10^{1}
var kon ge af Dan mark fra 1906 til 1912 . Han var det 230	1.136×10^{1}
v lig ut s ikt Aver Lipnos j An . Mys ig och barn	1.018×10^{1}
kevAl retro fArgadefArdenkAnslige . Barn	1.007×10^{1}
Det kan fin nas fl er pro jek t dArLars Bank vall med ver	9.832
v An lig at mos f Ar och sk A na rum , dock kans	9.703
. Det g l der de fl este v arer , her under l sk	9.656
L A~ ks et ind en 12 42 m 10 69 m 119 1 m	9.561
pA Chal mers fAr att kun na se dem . Sometimes I like to add	9.513
jeg g ings , stock ings well anything that covers my legs , is	9.509

(b) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 23581. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 2755 tokens, and has a mean of 2.578×10^{-1} (rank 0 for the output SAE latent) and a standard deviation of 2.428×10^{-2} over its non-zero values.

Example tokens	Max. activation
. \n400 Pet erm ans Bj erg 2940 m1200 m2940	7.399
from Sig urd . I think we were lied to . Don ' t you have	7.343
d "was invented by veter in arian Sig urd Ke il gaard in the	6.987
us Met z Ped ers en the film stars S ver rir G ud n	6.918
began working as an artist and teacher . \n In g ib j Ar g	6.917
ALE V intage Metal Ch airs From B ali By Bj A r k heim With	6.837
house , making Read Also : In grid N ils en Bio , D ating	6.760
film shot in Trin idad in 2013 , work by A y v ind F	6.647
og R oms dal , in Norway . \n484 Lav ang st ind en	6.632
K j ell ! \n I intended the first piece to basically be an introduction	6.463
world around me . Cong ratulations to Tor stein H org mo , the latest	6.459
and modern technology . \n As Tar je N issen – M eyer writes :	6.413

(c) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 48028. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 3567 tokens, and has a mean of 2.417×10^{-1} (rank 1 for the output SAE latent) and a standard deviation of 1.944×10^{-2} over its non-zero values.

Example tokens	Max. activation
., Netherlands , Italy , Canada , Denmark , Norway , and Turkey) and	9.613
EU , Germany , Norway , US and UK). \n PROMAN has been	9.009
Alaska and the Ale ut ian Islands , Green land , Britain , Norway ,	8.417
), Russia , Norway and Iceland which promotes dialogue , practical cooperation and development .	8.119
Sah ara . So have human rights bodies from Norway and elsewhere . \n The	8.066
the matern ity leave in Norway (46 weeks), Denmark (52 weeks)	8.006
W oc hen ende oder an Fe iert agen . THE Norwegian Fisher y Council	7.970
lands and islands and through the Norwegian f j ords and will continue to operate	7.870
single market by following Norway ' s model by joining the European Economic Area .	7.853
can either subs ist without Norway or simply dead zone it by stacking Uk r	7.850
is a member of the Norwegian Visual Artists Association and the Young Artists ' Society	7.838
Children Norway has been awarded the service contract for the Support to the Education Sector	7.760

(d) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 11698. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 295 tokens, and has a mean of 1.344×10^{-1} (rank 2 for the output SAE latent) and a standard deviation of 6.924×10^{-3} over its non-zero values.

Example tokens	Max. activation
kar , men du mAste vara in log g ad som an st Al ld	1.023×10^{1}
kA pa Euro och s Al ja US Dollar . \n Results per trade binary	9.895
l add ning och k lar att k opp las till slut st eg eller	9.721
ke vAl retro f Ar g ade fAr den kA ns lig e . Barn	9.363
pA Chal mers fAr att kun na se dem . Sometimes I like to add	9.343
v An lig at mos f Ar och sk A na rum , dock kans	8.634
och fAr fAr st Ar k are , upp till 20 tim mar ut an	8.587
C ancel . E tt ex emp el pA fore x hand el Ar att	8.477
dd rar mell an oli ka produ k ter . Dev ices and systems on	7.977
A till A ! Cong rat z Eva Ny strAm and Ad riel Young !	7.812
Det kan fin nas fl er pro jek t dAr Lars Bank vall med ver	7.368
pool och le k pl ats ! Tre v lig personal . \n– ub	7.195

(e) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 22804. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 895 tokens, and has a mean of 1.342×10^{-1} (rank 3 for the output SAE latent) and a standard deviation of 9.149×10^{-3} over its non-zero values.

Example tokens	Max. activation
so i fixed it myself but thats the first place you will get rust .	5.811
Models for Rust ing T ail gate Str uts . \n Tak ata recalls at	5.395
because you gave in easily Rust ic guys country a woman who is very to	5.063
bring to full resolution . Graphics are from a Rust ic W re ath collection	4.752
Rust ic Pic nic Table Popular Out door Furn iture Hand made By App al	3.102
we present . R ust ic Pic nic Table Amazing 31 All uring Ideas Tables	2.680
seeing . I have it in rust y pots and here it is in an	2.359
yields a Future , gen . cor out ine schedules the generator to be resumed	2.104
operations in order to ensure that the resulting LLVM IR can be ing ested	1.849
with @ gen . cor out ine . \n In Python 2 , the sub	1.805
from ‘ ()<> ‘. The ‘ / ‘ has no meaning besides being a	1.739
you will have to upload the . pub file and copy its contents to authorized	1.702

(f) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 12754. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 7 tokens, and has a mean of 1.168×10^{-1} (rank 4 for the output SAE latent) and a standard deviation of 5.760×10^{-3} over its non-zero values.

Example tokens	Max. activation
Turkey the League B victory . \n Swed en , making its Nations League debut	1.362×10^{1}
68 m 10 38 m 12 18 m HP on the Norwegian– Swed ish	1.330×10^{1}
for Lat v ian independence , R iga has been ruled by Germans , Swed	1.286×10^{1}
bar . \n summer when the Swed e began to shine . \n C up	1.180×10^{1}
en , Al Un ser , Swed e Savage , Bobby Un ser , Gordon	1.124×10^{1}
25 ppm in Denmark , France , Spain , Sweden and UK . \n "	1.107×10^{1}
\n This study was supported by the Swedish Research Council (Grant K 2015–	1.090×10^{1}
Canada , Finland , Italy , Sweden and the UK . In the next few	1.088×10^{1}
ra ps in English , French , Spanish , Swedish and Mand arin . You	1.076×10^{1}
, the Middle East and Southeast Asia . \n In reality , the Swedish and	1.070×10^{1}
this threat is found from different countries such as Sweden , Malaysia , India ,	1.040×10^{1}
iron ore mine in the world . It ' s owned by the Swedish government	1.020×10^{1}

(g) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 30912. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 175 tokens, and has a mean of 1.133×10^{-1} (rank 5 for the output SAE latent) and a standard deviation of 6.893×10^{-3} over its non-zero values.

Example tokens	Max. activation
ga , 2012 ClA ir se ach n A3 PAN ob ,	8.579
la ri Paul Berg ag us Walter Gilbert . \n Dh ' ain mic	8.236
A j ir ag us Spar An a b h fu il lu ach aH	8.205
a l A3 d A il ar an su AN om h . TA	7.989
. An san n AN m A3 r don i om ath A3 ir 5	7.893
ire adh Ch un Ce oil ag us tr AN R na G . Is	7.412
m A nd (" mind , reason "). Related to Old English my nt an	7.394
2013 na u irl is AN g iol ca igh mi ot ail (	7.251
. \n Import o de k rom program o estas fac ila . \n A	7.154
\n A I iu event o lig i A L as al la orig ina	7.080
A il te ! St air na h Ai ire ann will take you on	7.046
b ild oj de event oj k aj b ild oj de k ateg orio	7.012

(h) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 57769. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 321 tokens, and has a mean of 8.286×10^{-2} (rank 6 for the output SAE latent) and a standard deviation of 1.343×10^{-2} over its non-zero values.

Example tokens	Max. activation
iGe eIKe i , i i iL~	2.700
i HI iLIJ i l l i i i l e I	2.051
do that ? \n1 : 10 : 38 KEVIN : If you	1.881
e~e3 ei eIK i l ii iLl i i l	1.822
i iL~ ik14 e IJ el , e i iL14 ikl	1.763
L i ik~e ei . e I i Il eL	1.637
IIJ aI aoI !! \n aI a al~c2e~a I cli aI	1.592
e314 e IJ e g e I i g14 i l e	1.564
Bin is one of pictures that are related with the picture before in the collection	1.488
iLGe i iL14 ikl e3 e iL14 ikl i	1.467
sealed bearing . Now I feel kinda like a***** $$ for asking in	1.458
e L ik I ik ii l ik iLl i h I	1.416

(i) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 42113. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 3 tokens, and has a mean of 7.971×10^{-2} (rank 7 for the output SAE latent) and a standard deviation of 6.880×10^{-3} over its non-zero values.

Example tokens	Max. activation
and arthritis . \n Y og ar aj G ugg ulu : Ing redients include	1.332×10^{1}
Europe and Rest of World . Ing redients of this market are Sol vents .	1.315×10^{1}
\n 04 . Ing ros so , L io hn & Salv atore – Fl	1.298×10^{1}
an con el audio original en Ing les . El cost o de adm isi A3n	1.298×10^{1}
2 , 000 calorie diet . * Daily Value not established . \n Other Ing	1.245×10^{1}
Sm ooth ie , Six Ing red ient Sp ir ul ina Sn ack ies	1.201×10^{1}
. 100 Ing redients In A Pictures For Kitchen Number 79 Is Im possible !.	1.173×10^{1}
Ind io Pap ago Te pez coh u ite Cream 2 Oz Ing redients :	1.153×10^{1}
. Ma Ing alls had no cabinets or refrigerator and she raised five children and	1.137×10^{1}
under stood Ing red ient , With Rec ipes , Simply Rec ipes ' explanation	1.132×10^{1}
rop ur Ing redients is a global supplier of ingredients and services developed to create	1.065×10^{1}
Bah asa Ing gr is Sing kat is just about the image we ascertained on	1.045×10^{1}

(j) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 8827. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 21 tokens, and has a mean of 7.330×10^{-2} (rank 8 for the output SAE latent) and a standard deviation of 5.439×10^{-3} over its non-zero values.

Example tokens	Max. activation
to scholars in Oriental Studies , Med ieval Literature , and History , The Gh	9.650
ia world . Be part of a medieval revolution against the Word King and his	8.678
trying to make a living in a medieval town full of warriors , sor ce	8.207
a couple who play medieval and Renaissance instruments , sing , dance and celebrate at	8.198
Sem ikh ah was the rise of the medieval university , which began to issue	8.038
located in Med ieval Sp on Street , Cov entry . Mark Andrew offers an	8.015
armies of the Cal iph ates , through the action of bloody medieval battles ,	7.999
Ng aw ang Nam gy al , father and un ifier of medieval Bh ut	7.911
, I needed to know what are they ate in medieval northern England ? Lots	7.847
th Annual Award Win ning L ough rea Med ieval Festival takes place from the	7.761
or like the individual parts in a ' cycle ' of Med ieval mystery plays	7.681
id yll ic picture for the season . \n The medieval charm of Vil n	7.672

(k) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 21697. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 87 tokens, and has a mean of 6.570×10^{-2} (rank 10 for the output SAE latent) and a standard deviation of 6.127×10^{-3} over its non-zero values.

Example tokens	Max. activation
football of his career , Russell Wilson finished Monday night ' s win over the Minnesota	1.082×10^{1}
. ST . PA UL , Minn . (AP) – The Minnesota Department	1.068×10^{1}
the Minnesota Birth Center ; and D ian na J ol les , faculty at	1.060×10^{1}
the Minnesota Historical Society . \n Begin ning with the 1929 volume there is a	1.060×10^{1}
. \n The Winnipeg Row ing Club , the Minnesota Boat Club and the Saint	1.055×10^{1}
Miami Marl ins Milwaukee Brew ers Minnesota Tw ins . Kentucky Wild cats Win C	1.050×10^{1}
H . Allen . The lock out of the musicians at the Minnesota Orchestra has	1.042×10^{1}
Post started spec ulating this past week that the Minnesota Orche str al Association might	1.032×10^{1}
The Minnesota Wild have recalled defense man Ryan Murphy from A HL Iowa per a	1.032×10^{1}
An index to the Minnesota period ical collection in the South St . Paul Library	1.010×10^{1}
Marl ins Milwaukee Brew ers Minnesota . Add the United States of Baseball to your	1.003×10^{1}
Paul L ur line Row ing Club as the Minnesota and Winnipeg Row ing Association	1.002×10^{1}

(l) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 13110. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 61 tokens, and has a mean of 6.486×10^{-2} (rank 11 for the output SAE latent) and a standard deviation of 6.027×10^{-3} over its non-zero values.

Example tokens	Max. activation
, 2019 Posted in Official News Tags : Fl utter programming Leave a comment on	6.019
? \n If you ' re not familiar with the Fl utter programming language and	5.044
Your Business with Amazon F BA ? \n Why You Should Master Fl utter Programming	4.472
mater the sought – after Google Fl utter language . Let ' s begin now	3.056
Why You Should Master Fl utter Programming ? rub ber plants indoor rubber plant growing	2.706
ew ar Mah ots av will be celebrated from 8 th April to 10 th	1.795
compatible with the industry . \n Widget s for your website . Order them .	1.511
this place . Eleanor notices this upon first arriving , thinking to herself that "	1.489
of illness and health . \n Find out why Swift Skin and W ound is	1.488
, some patients feel a st inging sensation from the injection & think it isn	1.473
You can also use the Contact Information widget from the Form W idget s .	1.358
Col oring Pages . Image Source : h ou zz . com . Free Online	1.283

(m) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 26452. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 4 tokens, and has a mean of 6.316×10^{-2} (rank 12 for the output SAE latent) and a standard deviation of 6.164×10^{-3} over its non-zero values.

Example tokens	Max. activation
been made in extending Medicaid coverage to American Indians and Alaska N atives , the	1.224×10^{1}
percent of American Indian and Alaska Native children were enrolled in Medicaid or CH IP	1.224×10^{1}
uninsured rate for American Indian and Alaska Native children and families remain un accept ably	1.153×10^{1}
issues in the Park Service . \n Former Alaska governor and advisory board chairman Tony	1.147×10^{1}
and mindset for the Alaska Con ce aled Hand gun Perm it . T aught	1.106×10^{1}
popular appet izers , s oups and sandwiches . \n Loc ally caught Alaska seafood	1.082×10^{1}
is Canada , Alaska and Hawaii . Your mattress will be refund ed , however	1.081×10^{1}
for shipping quotes to other destinations such as Alaska and Hawaii . \n This product	1.079×10^{1}
Mo ose and Wolf hunt . \n We hunt the Alaska Yuk on mo ose	1.066×10^{1}
row se listings of Member users here at Alaska Fl irt that are associated with	1.064×10^{1}
Cherry , BL M ' s Alaska regional manager , said the agency was pleased	1.057×10^{1}
. The paint appeared to still be wet . The package includes Alaska Yuk on	1.045×10^{1}

(n) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 32153. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 100 tokens, and has a mean of 6.202×10^{-2} (rank 13 for the output SAE latent) and a standard deviation of 5.347×10^{-3} over its non-zero values.

Example tokens	Max. activation
Han over , Wake , and Fors y th have more than half of their	1.788×10^{1}
he told SMS F Ad vis er . \n Mr Fors y th explained this	1.643×10^{1}
arr ion " ' . Thanks to Anthony Fors of Absolute Clean for volunte ering	1.616×10^{1}
O read more into AT O IDs than they should ," Mr Fors y th	1.531×10^{1}
son Super Consult ing director Stuart Fors y th said shortly after the release of	1.517×10^{1}
web h omes fors ale . com help you find the Jacksonville , FL homes	1.051×10^{1}
uit your id ols ; fors ake your fond do ings ; and the promised	7.176
you , nor fors ake you " (He b . 13 : 5);	6.366
for comfort in my difficult situation . Do not fors ake me , Good Jesus	6.248
ume already led many to fors ake the temple , and hold her ordin ances	5.815
which needs to be confessed and fors aken . \n 2 . We profit from	5.704
be submitted by e– mail (sc anned to fors ik ring @ lease	5.697

(o) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 56394. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 8 tokens, and has a mean of 5.939×10^{-2} (rank 14 for the output SAE latent) and a standard deviation of 3.459×10^{-3} over its non-zero values.

Example tokens	Max. activation
Norwegian f j ords around St ord , Norway . \n Read more about our	1.093×10^{1}
250 m crossing under a f j ord between Krist ians und and A ver	1.089×10^{1}
coast in Turkey , cru ising the F j ords of Norway , etc ,	1.033×10^{1}
zones and sc ramble up to the top of the f j ord . There	1.001×10^{1}
60 m 16 10 m S ogn og F j ord ane , Norway .	9.988
lands and islands and through the Norwegian f j ords and will continue to operate	9.885
You will also experience a f j ord cruise on the mighty S og ne	9.601
f j 80 stud is just slightly larger , so all I had to do	9.451
steering box anyways , so I decided to convert to f j 80 tie	9.137
is 57 . 5 kg , S agne f j orden , Norway , in	9.027
\n 426 Al ks f j ord j A~ kel en 120 4 m 113	8.791
k j v Noah : November 29 , Ethical dile mmas occur when a situation	8.114

(p) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 36481. Paired with the output SAE latent with index 64386, the Jacobian element is non-zero for 36 tokens, and has a mean of 5.820×10^{-2} (rank 15 for the output SAE latent) and a standard deviation of 4.891×10^{-3} over its non-zero values.

Figure 19. The top 12 examples that produce the maximum latent activations for the output SAE latent with index 64386, and the input SAE latents with which the mean values of the corresponding Jacobian elements are greatest. The Jacobian SAE pair was trained on layer 15 of Pythia-410m with an expansion factor of $R = 64$ and sparsity $k = 32$. The examples were collected over the first 10K records of the English subset of the C4 text dataset with a context length of 16 tokens.

Example tokens	Max. activation
surge into the cathode , producing a current that does the all – important work	6.720
so that he avoids distribution of designated product that apparently does not meet legal requirements	6.606
, but also expansive alien environments that do their part to make the audience feel	6.584
of 2016 (88 percent) were opportun istic attacks that did not target a	6.467
to make therapeutic claims , were found to do so . \n Unfortunately the proposed	6.406
. Children are the most severely affected by poverty because they do not have the	6.331
hospital admissions ; emergency room visits that do not result in admission are excluded .	6.282
a court order . Pro pos als relating to children often do not need to	6.264
. S uitable exercises for pregnant women are those that do not strain the lower	6.255
? \n Unfortunately , the traditional Chinese approach to training in the modern world does	6.240
also request fat sh aming to be made illegal because it does not have any	6.232
select the 1 dB degradation to noise as the interference standard , since it does	6.221

(a) The top 12 examples that produce the maximum latent activations for the output SAE latent with index 60542. Output latent 60542 responds to a very specific use of the word "do": its use as a pro-verb. In a pro-verb a simple verb stands in for another more complex one and here "do" is a shorthand for an action that can only be understood from the context, for example, in "were found to do so" the "to do so" stands in for "to make therapeutic claims". Some of the inputs include very different uses of "do", one for example deals with the "Done" in "Donegal", an Irish county. However, another input includes a subtly different use of "do": cases where "do" is used as an auxiliary, modifying another verb, as in "[t]his does not meet the requirements". Clearly this circuit is creating a very fine distinction between different ways the word "do" can be used, a distinction we make in language comprehension, but one we would have trouble identifying or describing.

Example tokens	Max. activation
Your information will not be stored on HS BC ' s systems if you do	2.098×10^1
. Children are the most severely affected by poverty because they do not have the	2.077×10^1
surge into the cathode , producing a current that does the all – important work	2.056×10^1
. 001) and fewer over tri aged children who did not require inpatient management	2.043×10^1
, but also expansive alien environments that do their part to make the audience feel	2.032×10^1
us at your own cost within days of delivery . If you do not	2.027×10^1
them post menopausal . They were compared with more than 600 women who did not	2.014×10^1
despite the risk . When persecution came , they did not scatter . They remained	2.001×10^1
levels protect public health and welfare if they do not exceed 45 dB . \n	1.997×10^1
so that he avoids distribution of designated product that apparently does not meet legal requirements	1.996×10^1
hospital admissions ; emergency room visits that do not result in admission are excluded .	1.984×10^1
. S uitable exercises for pregnant women are those that do not strain the lower	1.976×10^1

(b) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 21465. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 13107 tokens, and has a mean of 2.529×10^{-1} (rank 0 for the output SAE latent) and a standard deviation of 5.392×10^{-2} over its non-zero values.

Example tokens	Max. activation
. We expect the market to do the opposite of what the indicators are saying	1.438×10^{1}
with the 2018 elections looming . \n They believe that Trump has done a good	1.313×10^{1}
only thing that she wanted in the world , but she did a bad thing	1.292×10^{1}
led by its President Jim Cor cor an , have done a wonderful job of	1.292×10^{1}
MB TA does so as well . And many state laws do the same .	1.264×10^{1}
a " t ote ; " I just buy the boxes .) These do a	1.261×10^{1}
can ' t win games , you ' re in trouble . \n Saint s did a	1.258×10^{1}
be ex orbit ant . You need do your own research to find out the	1.257×10^{1}
up for grabs , it ' s always Jews doing the grabbing ! \n Here	1.251×10^{1}
your muscles do the work , going at a sn ail ' s pace doesn ' t	1.246×10^{1}
to worry about whether the Black haw ks are doing the right thing with defense	1.244×10^{1}
You ' re doing the same thing on Amazon through sponsored ads and proving yourself	1.239×10^{1}

(c) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 61756. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 2683 tokens, and has a mean of 1.076×10^{-1} (rank 1 for the output SAE latent) and a standard deviation of 2.253×10^{-2} over its non-zero values.

Example tokens	Max. activation
A du grand lux e , il fait ce qu ' on lui dit ,	1.044×10^{1}
. Para mAs inform aciA3n , h aga cl ic en el la zo ab	9.095
cl ase de ing l As y por hacer la present aciA3n . A E	8.587
qu A h aces ? \n if you want to go back to a previous	6.694
si vous fa ite des codes ! \n l NGNh12 1 NI1	5.713
co , donde se real iza la seg unda esc ena y final mente ,	5.181
into the software . The K art ra interface is faire ly well designed for	4.361
which they draw drinking water . \n Leg isl ators must find the faire st	4.314
en do en . \n ' Living with Fran ', van af z ond ag	4.312
. De k ache ls m aken ve el law a ai . We ver	4.253
organis ieren of tm als ih re eigen en Games . Es tut uns le	4.225
e en p oner los hue v os en el interior del hu A sp	4.202

(d) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 51331. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 90 tokens, and has a mean of 8.908×10^{-2} (rank 2 for the output SAE latent) and a standard deviation of 9.883×10^{-3} over its non-zero values.

Example tokens	Max. activation
the middle of war again . \n The Seventh Done gal took part in the	1.341×10^1
all hurting tonight . I ' ll share details later this week . \n Done !	1.216×10^1
fit or there ' s something wrong please contact us . \n Done with Multi	1.211×10^1
this point . \n Done . Will be interested to learn more about this myself	1.192×10^1
the goal of Universal Male– Female Liter acy a done deal . \n No	1.191×10^1
feeling of being let down , overwhelmed and done in . Dis appoint ment is	1.191×10^1
there ' s something wrong please contact us ! \n Done with Egg– shaped	1.188×10^1
first gin distilled in Co . Done gal . D Ao lam An is the	1.179×10^1
Cl osure and re clamation of a mining property is a complex process . Done	1.168×10^1
they could take . \n The Done gal Guards , like no other arm of	1.155×10^1
were able to finish rebuilding efforts , and the Done gal Guards found themselves in	1.150×10^1
out his heart to the done ky . One day , K esh ava was	1.146×10^1

(e) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 11694. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 540 tokens, and has a mean of 7.039×10^{-2} (rank 3 for the output SAE latent) and a standard deviation of 1.967×10^{-2} over its non-zero values.

Example tokens	Max. activation
relax mechanisms . \n On the other hand , between the classic models , it	1.057×10^1
protein . For zinc prot op orph yr in , on the other hand ,	1.020×10^1
compared to Cauc as ians . On the other hand , the latter have a	9.974
all change associated with un ification . \n On the first point , see J	9.717
) the effect of class size varies across students . \n On the first form	9.667
disease . \n On the second day of hospitalization , B its y ' s	9.630
much scope for development . On the other hand , there is much scope for	9.605
for whom it is of minimal use . On the other hand , when hazards	9.602
regulates its collective temperature at different ambient air temperatures ; on the left side it	9.537
is . On the inside there are also lent icular films that diffuse the view	9.520
associated with these ex libr ises . On the one hand , one part of	9.519
states . On the other hand , we need to define our destinations in a	9.441

(f) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 48418. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 91 tokens, and has a mean of 5.105×10^{-2} (rank 4 for the output SAE latent) and a standard deviation of 9.043×10^{-3} over its non-zero values.

Example tokens	Max. activation
of the date of filing of the petition . \nA change of name or	5.590
the jury . \nAn interruption of the opposing counsel with a speech rather than	5.370
of these statutory changes . \nA new form , called a " Dis closure	5.359
. The Court will examine only marginally whether the principle is fulfilled . A detailed	5.215
filing of the petition . \nA merger is recorded by means of a petition	5.214
had committed a crime while out on release : obstruction of justice . \nA	4.802
does not accept liability for failure to deliver within the stated time . \nA	4.802
igated sentencing . An experienced Mar ic opa County D UI Attorney will know what	4.660
169 of the Revised Statutes . \nAn te cedent : Ref erring to a	4.463
trial ; judge ' s availability limited . \nA second discrimination trial against the University	4.404
a grounds for dealing with the issue . A pharmacy is also nearby which sells	4.393
to comply with its requirements . The court noted that "[a] d vance	4.323

(g) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 32517. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 1 tokens, and has a mean of 5.015×10^{-2} (rank 5 for the output SAE latent) and a standard deviation of 0.000 over its non-zero values.

Example tokens	Max. activation
photos . Import ant : This product does not include the techniques used in the	1.195×10^{1}
p umpyou are looking for . \n This product does not contain 1 ,	1.182×10^{1}
Furthermore , exp or ail . org and web sites associated does not sell user	1.146×10^{1}
. \n The name T ee berg does not appear on any of the top	1.130×10^{1}
suggestion for ASC Utt imate . \n Do you do not need to worry about	1.123×10^{1}
ube . com Limited does not endorse any user submission , and expressly disclaim s	1.119×10^{1}
ube . com Limited does not permit copyright infring ing activities or infringement of intellectual	1.119×10^{1}
' s family , it doesn ' t for Mark he told Ost rov	1.095×10^{1}
ig ences . com website or webpage , Dil ig ences does not represent or	1.087×10^{1}
market , it does not help anyone no mater how great the thought is .	1.085×10^{1}
marketing , Blog ging , internet marketing \t . \n The world doesn ' t	1.075×10^{1}
a daily basis . \n Hey , I don ' t know if I ' m posting	1.075×10^{1}

(h) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 23968. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 2561 tokens, and has a mean of 4.760×10^{-2} (rank 6 for the output SAE latent) and a standard deviation of 1.204×10^{-2} over its non-zero values.

Example tokens	Max. activation
you can lower your standard of living . And a lower standard of living does	9.428
? Of course not . A post hum ous diagnosis does not change who he	9.376
holds equity in a private business or has a large portfolio this does not	9.172
that this is a negative review , the above observations do constitute qu ib bles	9.046
Tax ote re and permanent hair loss , those warnings do not appear on any	8.894
have no responsibility or control over them . The existence of these links does not	8.834
and ambitious . \n M ission success does not require these complex maneu vers ,	8.667
. Please note these restrictions do not apply to exhib itors . \n To help	8.648
Yellow stone if there are only that many sub species . That does not include	8.556
But the Tribunal explained , " Training alone does not meet the requirements of due	8.529
\n Some schools prefer to be data controllers . This does make it easier for	8.483
extreme heat . It ' s naturally darker colour doesn ' t do itself any	8.473

(i) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 19973. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 2691 tokens, and has a mean of 4.681×10^{-2} (rank 7 for the output SAE latent) and a standard deviation of 9.146×10^{-3} over its non-zero values.

Example tokens	Max. activation
play and revenue . Well , that no longer appears to be the case .	8.329
than he believes or cares to say . Poll s now demonstrate this to be	8.314
systems are often thought to be much more securely protected than is actually the case	7.874
relations with residents . Unfortunately , with some boards this is not always the case	7.272
some cases the spam detection stopped working . This is no longer the case .	7.104
' t) and if there are no clear winners between them (which is usually	7.036
found the same to be true for them !). Ch ur ro Fried Ice Cream	6.854
. \n However , even if that is not correct , the Court erroneously uses	6.826
final . However , that ' s not always the case . There are plenty	6.787
if that were not bad enough for Thor pe ' s membership , in the same	6.773
the lease period has expired and no oil has been produced . If this is	6.688
or missing from the root of the domain you added . If this is the	6.680

(j) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 13058. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 431 tokens, and has a mean of 4.528×10^{-2} (rank 8 for the output SAE latent) and a standard deviation of 1.961×10^{-2} over its non-zero values.

Example tokens	Max. activation
Compan ies must give workers the information they need to perform their jobs safely ,	1.448×10^1
too short to drink bad coffee that makes you perform poorly and feel awful .	1.448×10^1
of your reserve fund . By performing a reserve study , you can determine how	1.426×10^1
g mail account , perform my edit , and then un install in a matter	1.410×10^1
worn fill at the tail pipe as permitted to perform every engine and every rate	1.409×10^1
s car just got away from him and he performed a lazy spin into the	1.400×10^1
from , modify , publish , edit , translate , distribute , perform , display	1.400×10^1
The firm did not perform impurity testing during stability on two products since 2016 .	1.397×10^1
stop thinking about mess ing up . You performed so tent atively that you hoped	1.389×10^1
like how black hat hackers are incent iv ized by money to perform malicious activities	1.388×10^1
and payments encourage P BM s to actually perform to their contractual guarantees instead of	1.367×10^1
individual educational plans ; assists in inventory control ; and performs other duties related to	1.364×10^1

(k) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 56700. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 574 tokens, and has a mean of 4.375×10^{-2} (rank 9 for the output SAE latent) and a standard deviation of 1.275×10^{-2} over its non-zero values.

Example tokens	Max. activation
share the posts as if your life depended on it . Which it doesn ' t	8.326
encounters an impossible reality that shouldn ' t exist , but does . \n A	8.273
ask ! Who knew las agna was for dessert ? We sure didn ' t	7.129
to make sure that it works correctly and it does . \n What '	6.960
, status and aside . Not every WordPress theme supports post formats and those that	6.757
you ever thanked me ? \n no you actually did , but I can count	6.578
if this would kick in , but it didn ' t . \n After installing	6.342
don ' t need convincing . But do you know who does ? \n And	6.318
big one . It seems to work . Or does it ? Then three fish	6.302
. It seems to work . Or does it ? Then three fish get the	6.277
Spin ! You dont switch but your opponent does so you can stack damage on	6.275
Some how I envisioned our family as a family of four . I still do	6.228

(l) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 46097. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 3589 tokens, and has a mean of 3.995×10^{-2} (rank 10 for the output SAE latent) and a standard deviation of 2.127×10^{-2} over its non-zero values.

Example tokens	Max. activation
its spark . Employees are not inspired , stakeholders are becoming color less and you	5.325
are talking about are humans , and not robots , and so a greater amount	4.681
nice " not too fast , not too slow " rhythm and settled in to	4.430
end up being about communication not between professions , but with clients and families	4.387
pre– s lic ed is easiest not too thin , and nothing wet	4.313
, I did not know how the French health care worked and I did not	4.230
move frequently , not be on school district census rolls , and are less likely	4.183
Total PM . A way of life . Not a job . \n It was	4.141
the fire which does not burn , the water which does not wet the hands	4.113
I do not move , I do not breath . I close my eyes and	4.075
It was not for a meeting with an international leader . \n And it was	4.069
such thing as safe plastic aG not for you and your family , and not	4.018

(m) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 4510. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 133 tokens, and has a mean of 3.978×10^{-2} (rank 11 for the output SAE latent) and a standard deviation of 1.006×10^{-2} over its non-zero values.

Example tokens	Max. activation
witnessing before moving on to more difficult or resistant areas . Rather the opposite was	1.143×10^{1}
decision making authority . Actually , the opposite is implied . Let me put it	1.059×10^{1}
should come before focusing on one ' s own , and I feel the reverse	1.048×10^{1}
t faith . It ' s actually the opposite of hope . Our other options	1.017×10^{1}
tourist – rich locale , but rather , the opposite is true . El is	1.004×10^{1}
good guy ; if anything the opposite was said when saying Zimm erman ' s	9.978
out of fear mode , you return to love mode . The opposite of love	9.799
in any way–– just the opposite . The fine balance between supp len ess	9.498
. process as perform argues the opposite : that learn on method must take heavily	9.484
says the right words to us , but he acts just the opposite . He	9.464
' s a bad thing , in this case , it means the opposite .	9.423
– EGR– val ve Without voltage applied , it ' s just the opposite	9.166

(n) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 28695. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 53 tokens, and has a mean of 3.952×10^{-2} (rank 12 for the output SAE latent) and a standard deviation of 7.511×10^{-3} over its non-zero values.

Example tokens	Max. activation
\n I ’ ve got to admit , I ’ ve heard some " do oz ie	8.284
silent film . Google ’ s ever changing " google do od le " logo	7.809
our annual celebrations can be . Do pp el k lic ke den Number Sl	7.234
3 . \n Here ’ s another do oz ie : Both Av ig dor	7.154
doing a few do od les of people in suits with their arms crossed .	6.835
the institutions tend first information in using the has hers , print able do od	6.828
ee F aret . This was long before the Do ob ie Brothers ever thought	6.275
keeping were a part of our life and I wanted to know about Do ob	6.262
an object I do remember is Do ob ee F aret . It was a	6.177
adopt me . Name on birth c irt ificate was baby do y j ohn	6.137
found that do od lers performed 29% better than non– d ood lers	5.989
a do v ish Fed and institutional instability in the US ," said Kyle Rod	5.951

(o) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 26469. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 29 tokens, and has a mean of 3.831×10^{-2} (rank 13 for the output SAE latent) and a standard deviation of 1.040×10^{-2} over its non-zero values.

Example tokens	Max. activation
Sp an ien k auf en , k Annen Sie eine span ische Auf enthal ts	8.197
d abe i auch auf Sport w et ten , die man auch be qu	8.086
acht eine Ab z oc ke , da her w A14r de ich r A14ck	8.018
k auf en will , kann ich tro tz dem das Gold V isa be	7.818
ft wer de ich A fter im Dr A14ck Gl A14ck Online Casino an Turn	7.550
and und mehr ! \n Fr age : K ann ich ein span is ches	7.512
sind . \n Fr age : K ann ich eine Fin anz ierung er hal	7.413
, dass es ges etz lich an erk ann te An ge h Ar ige	7.390
K ann ich nur we ite rem p fe h len ." \n " This	7.366
we isen , dass Sie dire kt oder ind ire kt In hab er der	7.351
we isen , dass Sie nicht mehr als 11 Mon ate a uss er hal	7.351
age : W enn ich die Imm ob il ie durch eine Ges ells chaft	7.225

(p) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 14813. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 2 tokens, and has a mean of 3.811×10^{-2} (rank 14 for the output SAE latent) and a standard deviation of 8.688×10^{-3} over its non-zero values.

Example tokens	Max. activation
. \n4 Does the Pale o Diet E lim inate Healthy Foods Like Brown	3.101
for all for profit show tickets as required by Louisiana law . \n Does not	3.047
0334 . \n Does my infant / child need a ticket for the show	2.843
000 . \n Does CODE undertake its own due diligence ? \n Yes , should	2.575
duties . \n Does this mean we are moving to a system where the Pope	1.695
am ore ; Ronald Bra ut ig am , p f ; Leo van Does	1.543
. \n Does n ' t clog pores nor does cause any acne break out	1.511
. \n Does he have a lot of toys ? Yeah , can we bring	1.511
. Does Ben Bishop fix their go alt ending ? Where do Alexander Rad ul	1.479
. Does this mean that the leaders who engage in the methods are bad people	1.479
. Does yours look as pretty as mine ? \n I ' m going to	1.479
day . \n Does your marketing team regularly grapp le with problems ? If so	1.306

(q) The top 12 examples that produce the maximum latent activations for the input SAE latent with index 41425. Paired with the output SAE latent with index 60542, the Jacobian element is non-zero for 17 tokens, and has a mean of 3.734×10^{-2} (rank 15 for the output SAE latent) and a standard deviation of 8.725×10^{-3} over its non-zero values.

Figure 20. The top 12 examples that produce the maximum latent activations for the output SAE latent with index 60542, and the input SAE latents with which the mean values of the corresponding Jacobian elements are greatest. The Jacobian SAE pair was trained on layer 15 of Pythia-410m with an expansion factor of $R = 64$ and sparsity $k = 32$. The examples were collected over the first 10K records of the English subset of the C4 text dataset with a context length of 16 tokens.

D. Training

Our training implementation is based on the open-source SAELens library (Bloom et al., 2024). We train each pair of SAEs on 300 million tokens from the Pile (Gao et al., 2020), excluding the copyrighted Books3 dataset, for a single epoch. Except where noted, we use a batch size of 4096 sequences, each with a context size of 2048. At a given time, we maintain 32 such batches of activation vectors in a buffer that is shuffled before training, which reduces variance in the training signal.

We use the Adam optimizer (Kingma & Ba, 2017) with the default beta parameters and a constant learning-rate schedule with 1% warm-up steps, 20% decay steps, and a maximum value of 5×10^{-4} Additionally, we use 5% warm-up steps for the coefficient of the Jacobian term in the training loss. We initialize the decoder weight matrix to the transpose of the encoder, and we scale the decoder weight vectors to unit norm at initialization and after each training step (Gao et al., 2024).

Except where noted, we choose an expansion factor $R = 32$, keep the $k = 32$ largest latents in the TopK activation function of each of the input and output SAEs, and choose a coefficient of $\lambda = 1$ for the Jacobian term in the training loss.

D.1. Training signal stability

We initially considered the following setup:

$$\mathbf{s}_{\mathrm{x}} = e_{\mathrm{x}}(\mathbf{x}) \,, \quad \hat{\mathbf{x}} = d_{\mathrm{x}}(\mathbf{s}_{\mathrm{x}}) \,, \quad \mathbf{y} = f(\hat{\mathbf{x}}) \,, \quad \mathbf{s}_{\mathrm{y}} = e_{\mathrm{y}}(\mathbf{y}) \,, \quad \hat{\mathbf{y}} = d_{\mathrm{y}}(\mathbf{s}_{\mathrm{y}}) \tag{32}$$

The problem with this arrangement is that the second SAE depends on an output from the first SAE. Since both SAEs are trained simultaneously, we found that this compromised training signal stability – whenever the first SAE changed, the training distribution of the second SAE changed with it. Additionally, at the start of training, when the first SAE was not yet capable of outputting anything meaningful, the second SAE had no meaningful training data at all, which not only made it impossible for the second SAE to learn but also made the first SAE less stable via the Jacobian sparsity loss term.

To address this problem, we instead used this setup:

$$\mathbf{s}_{\mathrm{x}} = e_{\mathrm{x}}(\mathbf{x}) \,, \quad \hat{\mathbf{x}} = d_{\mathrm{x}}(\mathbf{s}_{\mathrm{x}}) \,, \quad \mathbf{y} = f(\mathbf{x}) \,, \quad \mathbf{s}_{\mathrm{y}} = e_{\mathrm{y}}(\mathbf{x}) \,, \quad \hat{\mathbf{y}} = d_{\mathrm{y}}(\mathbf{s}_{\mathrm{y}}) \tag{33}$$

Importantly, we pass the actual pre-MLP activations $\mathbf{x}$ rather than the reconstructed activations $\hat{\mathbf{x}}$ into the MLP f. In addition to improving training stability, we believe this setup to be more faithful to the underlying model because both SAEs are trained on the unmodified activations that pass through the MLP.

E. Evaluation

We evaluated each of the input and output SAEs during training on ten batches of eight sequences, where each sequence has a context size of 2048, i.e., approximately 160K tokens. We computed the sparsity of the Jacobian, measured by the mean number of absolute values above 0.01 for a single token, separately after training. In this case, we collected statistics over 10 million tokens from the validation subset of the C4 text dataset.

For reconstruction quality, we report the mean cosine similarity between input activation vectors and their autoencoder reconstructions, the explained variance (MSE reconstruction error divided by the variance of the input activation vectors), and the MSE reconstruction error.

For model performance preservation, we report the cross-entropy loss score, which is the increase in the cross-entropy loss when the input activations are replaced by their autoencoder reconstruction divided by the increase in the loss when the input activations are ablated (set to zero).

For sparsity, we report the number of 'dead' latents that have not been activated (i.e., appeared in the k largest latents of the TopK activation function) within the preceding 10 million tokens during training and the number of latents that have activated fewer than once per 1 million tokens during training on average.

Given an expansion factor of 64, $k = 32$, and a Jacobian loss coefficient of 1, i.e., fixed hyperparameters, we find that the reconstruction error and cross-entropy loss score are consistently better for the input SAE than the output SAE. Additionally, we find that the performance is generally poorer for the intermediate layers than early and later layers.

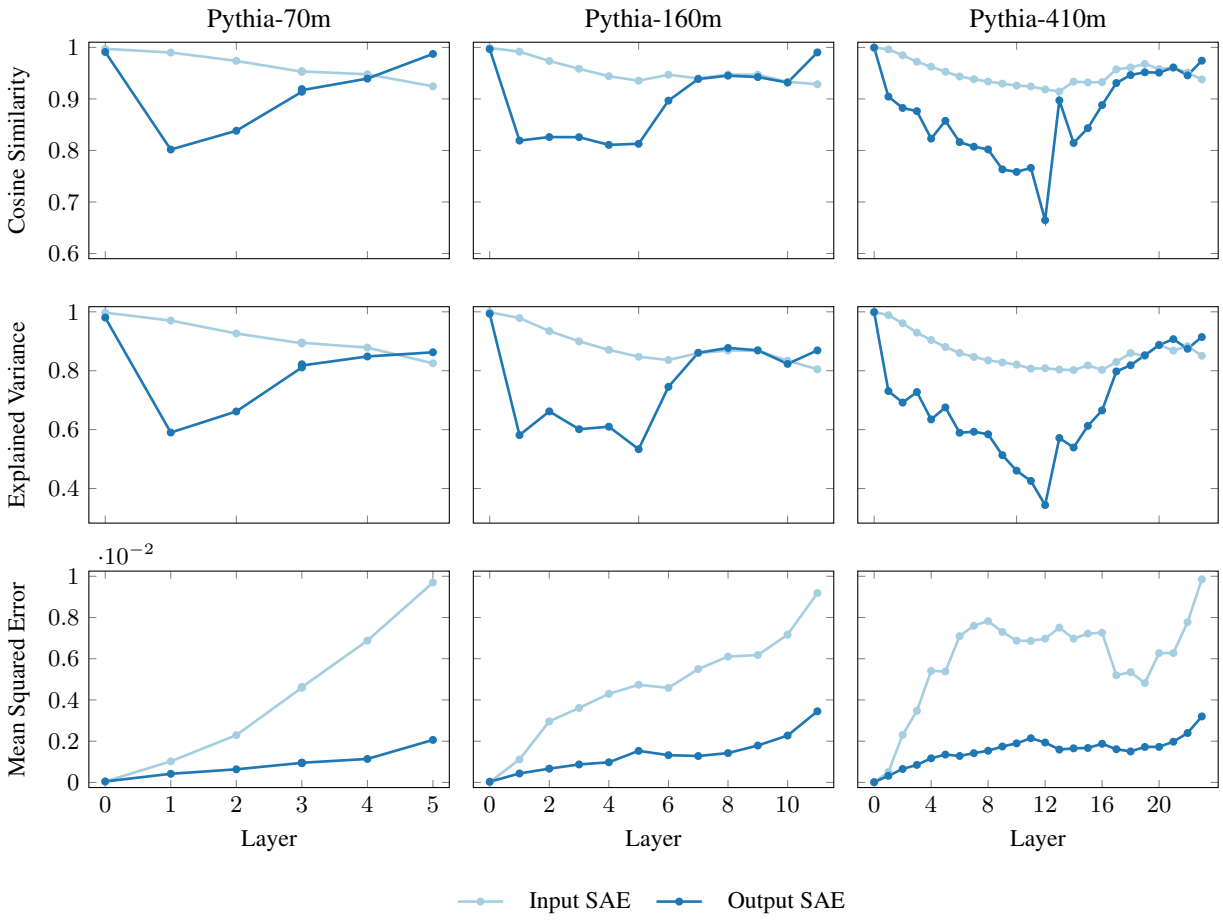


Figure 21. Reconstruction quality metrics for Jacobian SAEs trained on the feed-forward networks at every layer (residual block) of Pythia transformers. The cosine similarity is taken between the input and reconstructed activation vectors, and the explained variance is the MSE reconstruction error divided by the variance of the input activations. For each SAE, the expansion factor is $R = 64$ and $k = 32$; the Jacobian loss coefficient is 1.

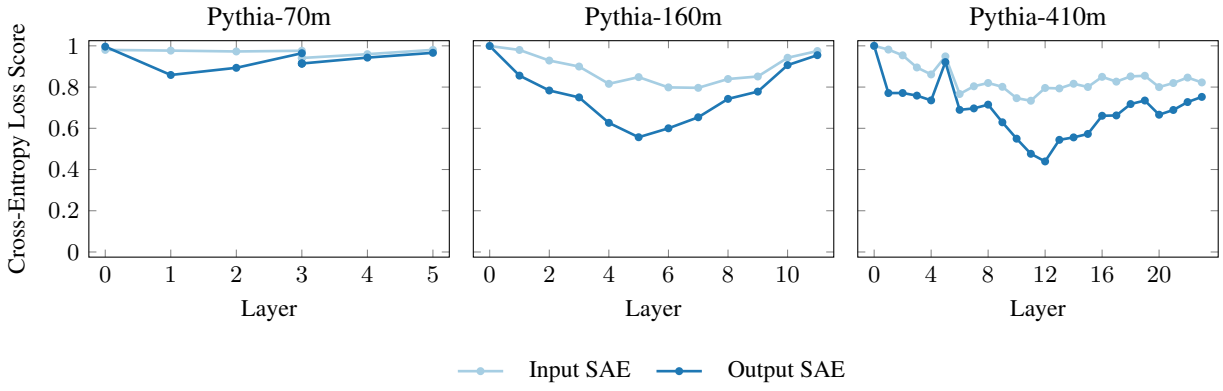


Figure 22. Model performance preservation metrics for Jacobian SAEs trained on the feed-forward networks at every layer (residual block) of Pythia transformers. The cross-entropy loss score is the increase in the cross-entropy loss when the input activations are replaced by their autoencoder reconstruction divided by the increase when the input activations are ablated (set to zero). For each SAE, the expansion factor is $R = 64$ and $k = 32$; the Jacobian loss coefficient is 1.

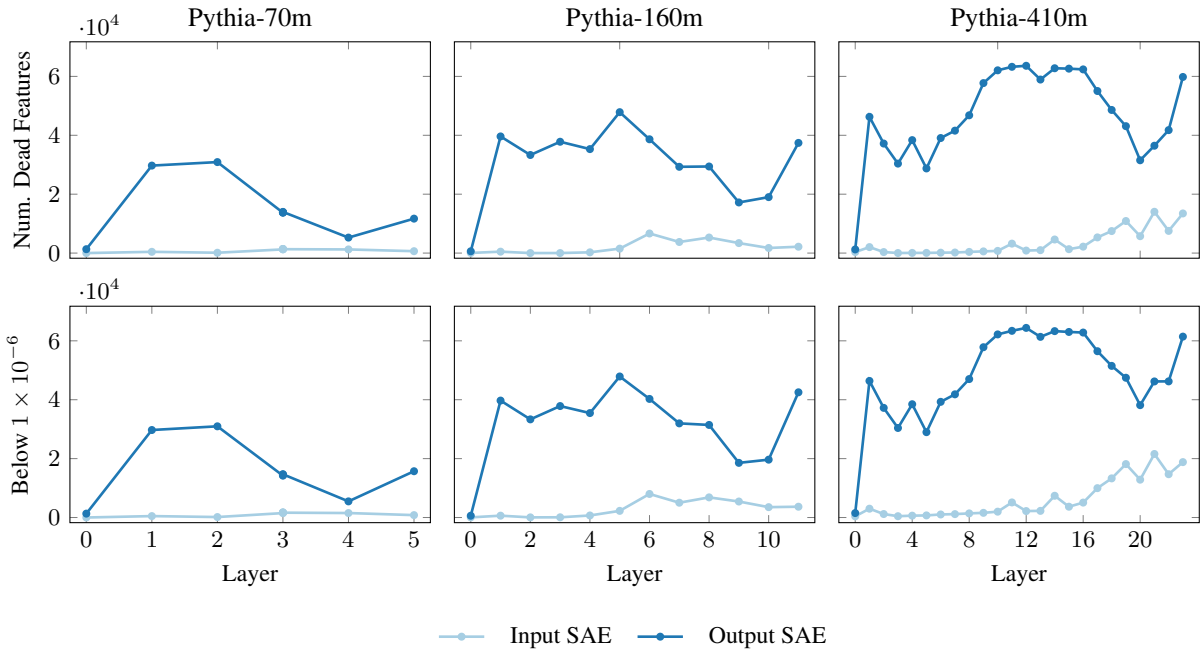


Figure 23. Sparsity metrics per layer for Jacobian SAEs trained on the feed-forward networks at every layer (residual block) of Pythia transformers. Recall that the L^0 norm per token for each of the input and output SAEs is fixed at k by the TopK activation function. For each SAE, the expansion factor is $R = 64$ and $k = 32$; the Jacobian loss coefficient is 1.

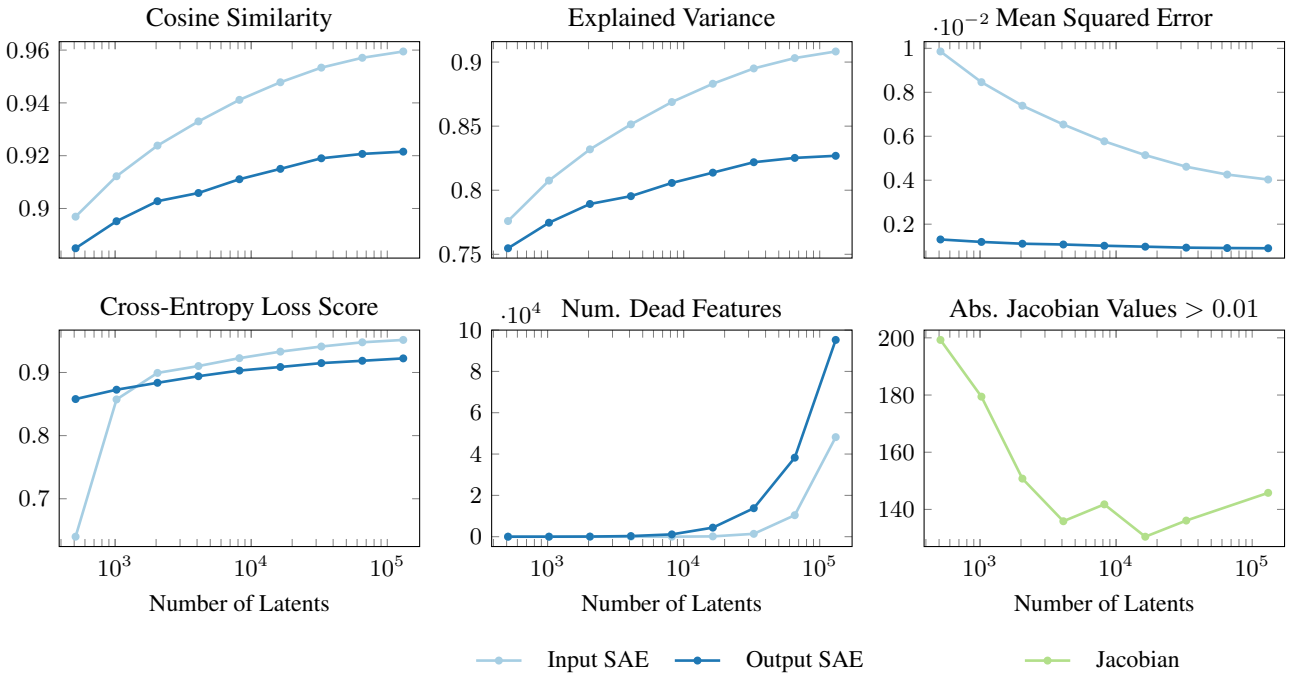


Figure 24. Reconstruction quality, model performance preservation, and sparsity metrics against the number of latents. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m (model dimension 512) with $k = 32$. Recall that the maximum number of non-zero Jacobian values is $k^2 = 1024$. The reconstruction quality and cross-entropy loss score improve as the number of latents increases, and the number of dead features grows more quickly for the output SAE than the input SAE. See Appendix E for details of the evaluation metrics.

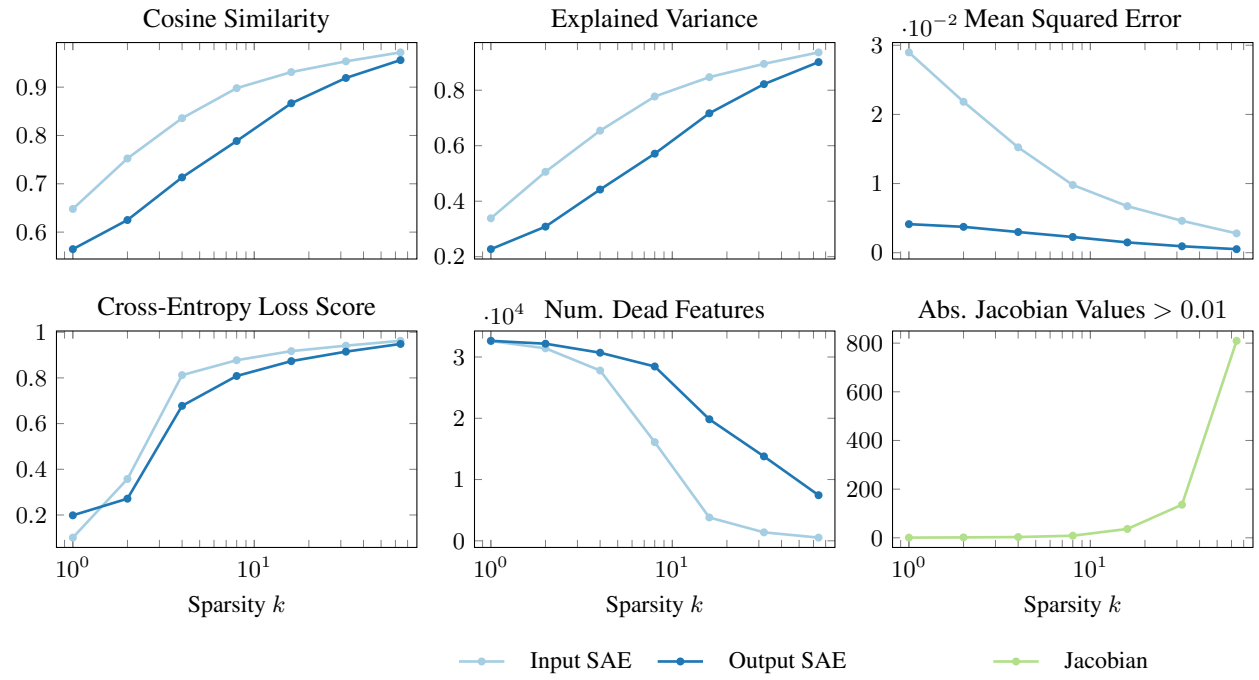


Figure 25. Reconstruction quality, model performance preservation, and sparsity metrics against the k largest latents to keep in the TopK activation function. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m with expansion factor $R = 64$. Recall that the maximum number of non-zero Jacobian values is k^2. The reconstruction quality and cross-entropy loss score improve as k increases, and the number of dead features decreases. See Appendix E for details of the evaluation metrics.

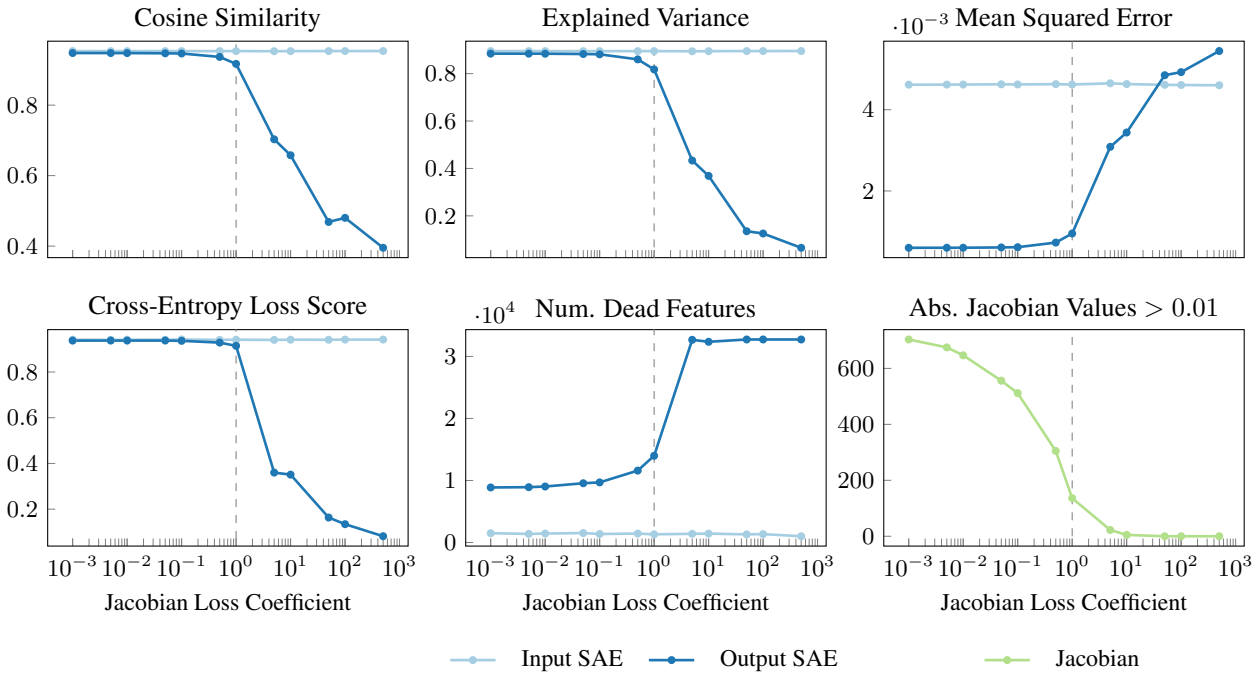


Figure 26. Reconstruction quality, model performance preservation, and sparsity metrics against the Jacobian loss coefficient. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m with expansion factor $R = 64$ and $k = 32$. Recall that the maximum number of non-zero Jacobian values is $k^2 = 1024$. In accordance with Figure 5, all evaluation metrics degrade for values of the coefficient above 1. See Appendix E for details of the evaluation metrics.

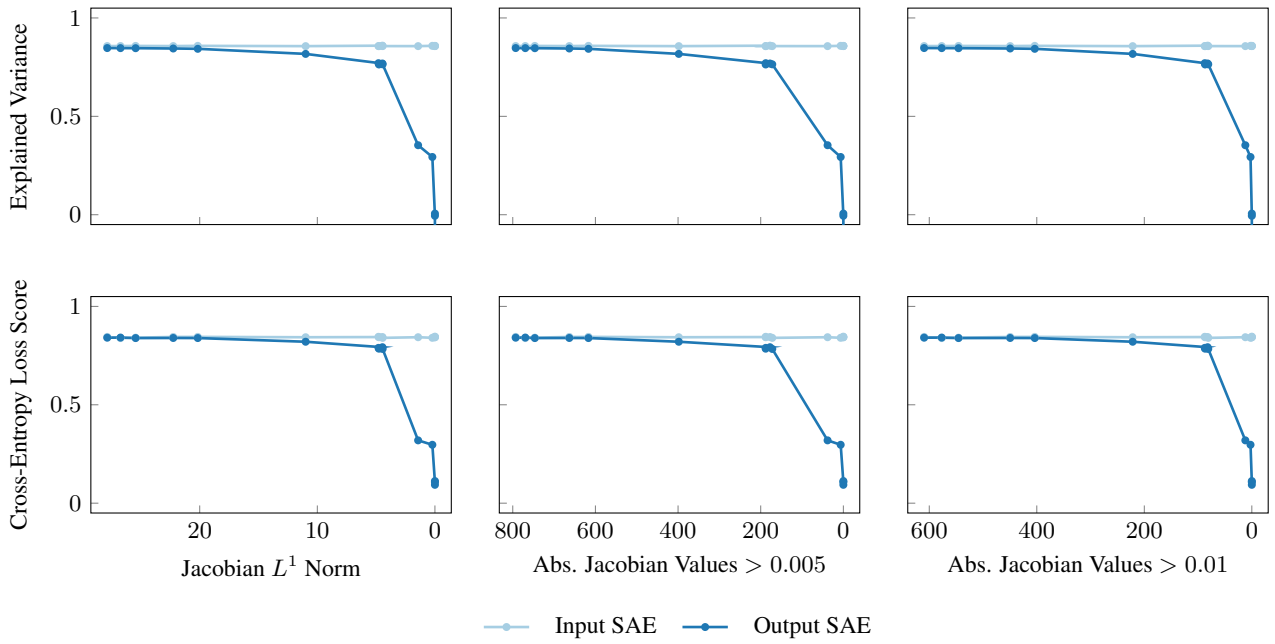


Figure 27. Pareto frontiers of the explained variance and cross-entropy loss score against different sparsity measures when varying the Jacobian loss coefficient. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 3 of Pythia-70m with expansion factor $R = 64$ and $k = 32$. Recall that the maximum number of (dead) latents is 32768 (64 times the model dimension 512), and the maximum number of non-zero Jacobian values is $k^2 = 1024$. See Appendix E for details of the evaluation metrics.

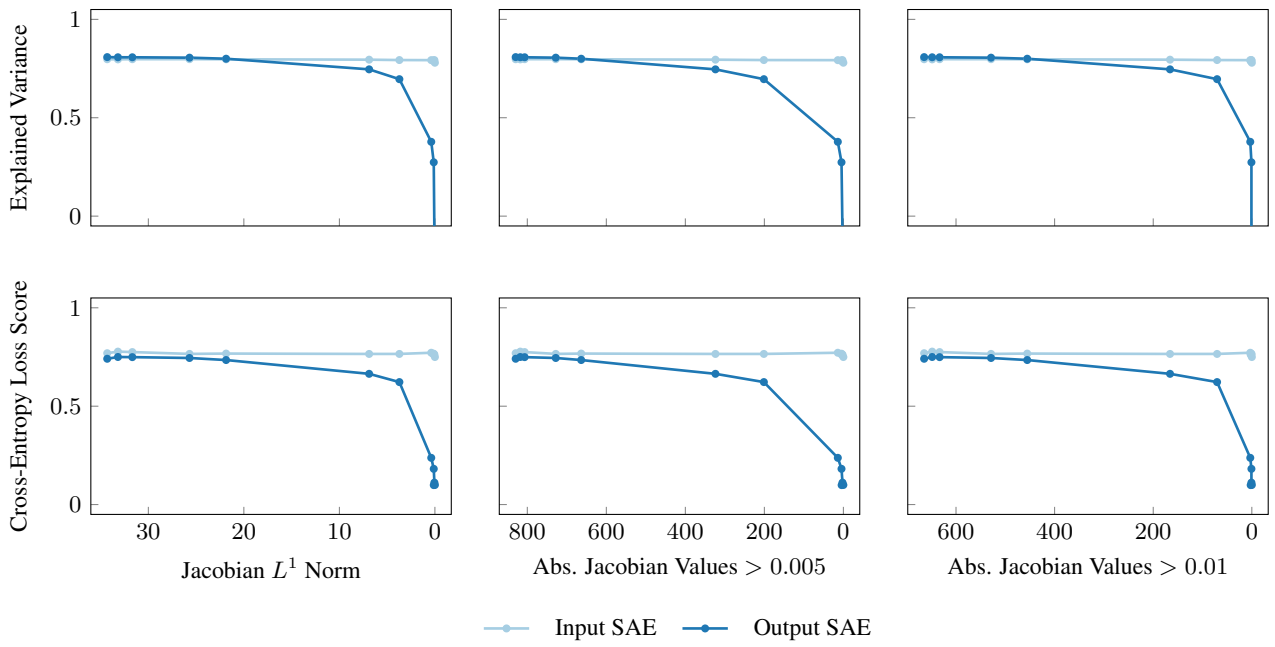


Figure 28. Pareto frontiers of the explained variance and cross-entropy loss score against different sparsity measures when varying the Jacobian loss coefficient. The coefficient has a relatively small impact on the reconstruction quality and sparsity of the input SAE, whereas it has a large effect on the sparsity of the output SAE and elements of the Jacobian matrix. Here, we consider Jacobian SAEs trained on the feed-forward network at layer 7 of Pythia-160m with expansion factor $R = 64$ and $k = 32$. Recall that the maximum number of (dead) latents is 49152 (64 times the model dimension 768), and the maximum number of non-zero Jacobian values is $k^2 = 1024$. See Appendix E for details of the evaluation metrics.

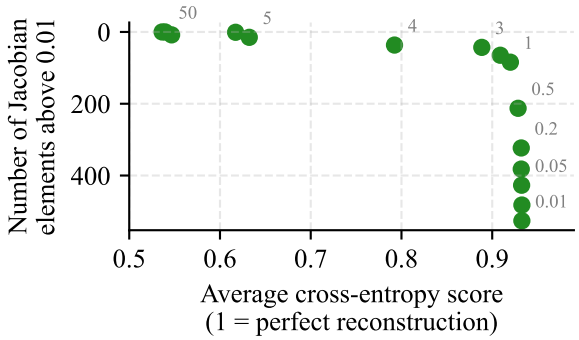


Figure 29. The trade-off between reconstruction quality and Jacobian sparsity as we vary the Jacobian loss coefficient. Each dot represents a pair of JSAEs trained with a specific Jacobian coefficient. Measured on layer 3 of Pythia-70m with $k = 32$.

We speculate that it is necessary to tune our hyperparameters for each layer individually to achieve improved performance; see, for example, Figures 26 and 4 for the variation of our evaluation metrics against the coefficient of the Jacobian loss term for individual layers of Pythia-70m and 160m.

F. More data on Jacobian sparsity

In Figure 23 we showed that Jacobians are much more sparse with JSAEs than traditional SAEs. To this end, we provided a representative example of what the Jacobians look like with JSAEs vs traditional SAEs. Some readers may object that this is not an apples-to-apples comparison since JSAEs are optimizing for lower L1 on the Jacobian, so it may be the case that JSAEs merely induce Jacobians with smaller elements, but their distribution may still be the same. To address this criticism, the examples are L2 normalized; we provide un-normalized versions as well as L1 normalized versions of the example Jacobians in Figure 30. We also provide a histogram and a CDF of the distribution of absolute values of Jacobian elements in Figure 32, which is taken across 10 million tokens.

F.1. Jacobian norms

In this section, we address an objection we expect some readers will have to our measures of sparsity. Our main metric for sparsity is the percentage of elements with absolute values above certain small thresholds (e.g. Figure 2). However, one can imagine two distributions with the same degree of sparsity, but vastly different results on this metric due to a different standard deviation. For instance, imagine two Gaussian distributions, both with $\mu = 0$ but with significantly different standard deviations, $\sigma_1 \gg \sigma_2$. They would score very differently on our metric, but their degrees of sparsity would not be meaningfully different (since sparsity requires there to be a small handful of relatively large elements). Since our L_1 penalty encourages the Jacobians to be smaller, it could be that they simply become more tightly clustered around 0. However, this is not the case. We can measure this by looking at the "norms" of the Jacobian, i.e. we flatten the Jacobian, treat it as a vector, and compute its L_p norms. If the Jacobian is merely becoming smaller, we would expect all of its L_p norms to decrease at roughly the same rate. On the other hand, if the Jacobian is becoming sparser, we would expect its L_1, L_2 norms to decrease while its $L_4, \ldots, L_\infty$ norms, which depend more strongly on the presence or absence of a few large elements, should stay roughly the same. We present these results in Figure 35, as we can see, the Jacobian does become slightly smaller, but most of the effect we see is indeed the Jacobian becoming significantly more sparse.

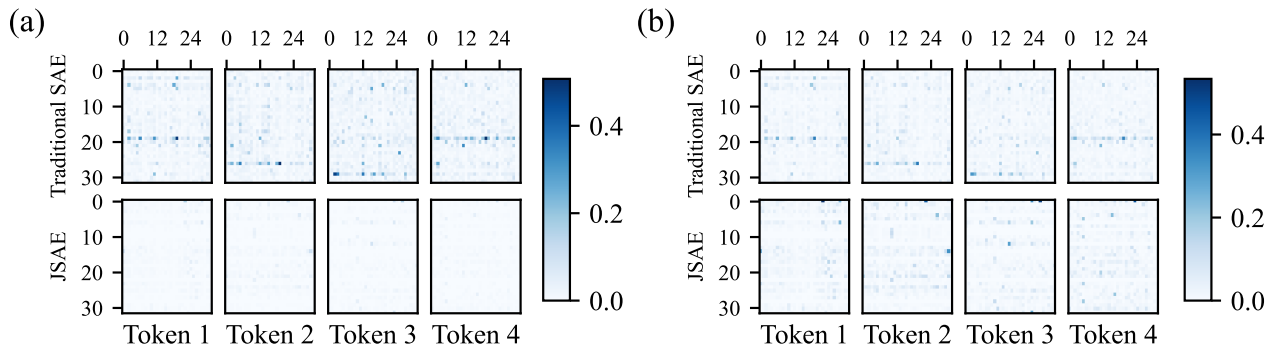


Figure 30. Comparison of Jacobians from traditional SAEs vs JSAEs, same as Figure 2 but with different normalization. (a) Not normalized. (b) L2 normalized. Measured on layer 15 of Pythia-410m.

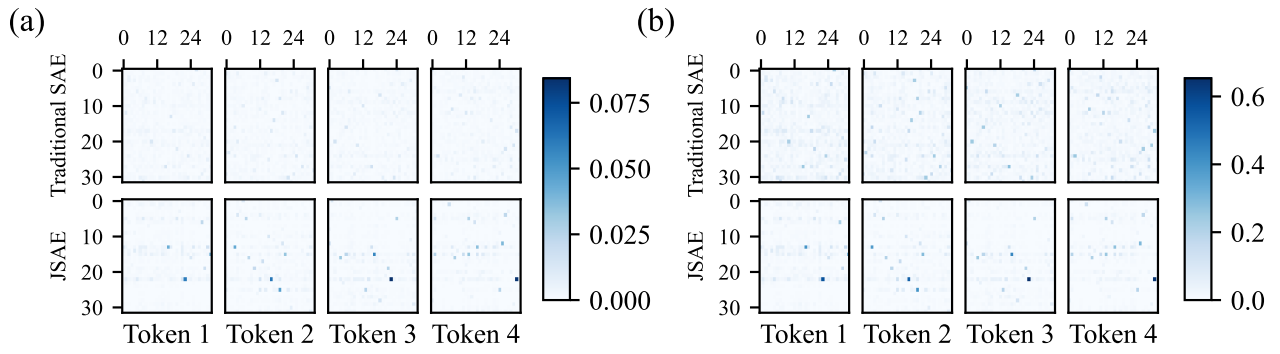


Figure 31. Comparison of Jacobians from traditional SAEs vs JSAEs, same as Figure 2 but with different normalization. (a) L1 normalized. (b) L2 normalized. Measured on layer 3 of Pythia-70m.

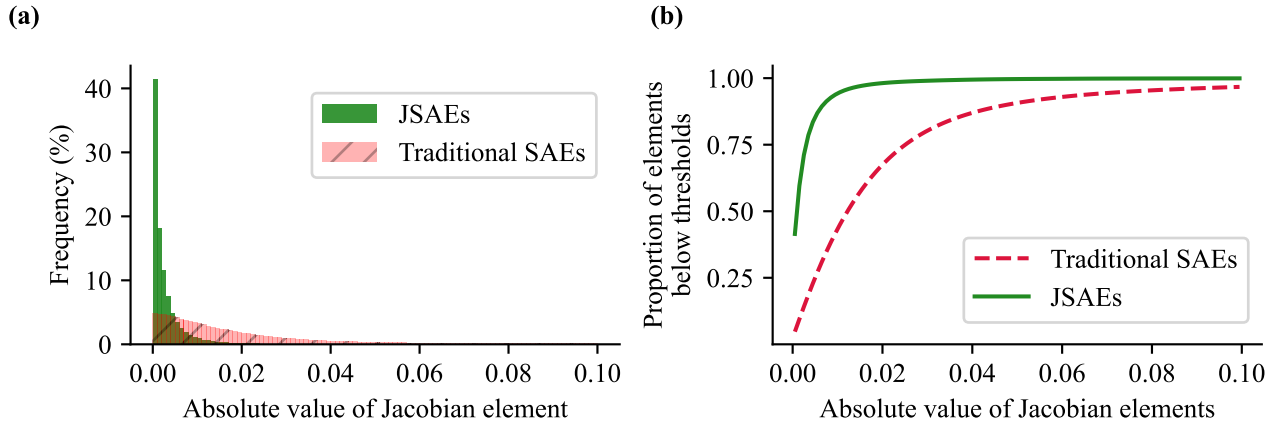


Figure 32. Further data showing that JSAEs induce much greater Jacobian sparsity than traditional SAEs. (a) A histogram of the absolute values of Jacobian elements in JSAEs versus traditional SAEs. JSAEs induce significantly more sparse Jacobians than standard SAEs. This means that there is a relatively small number of input-output feature pairs which explain a very large fraction of the computation being performed. Note that only the $k \times k$ elements corresponding to active latents are included in the histogram – the remaining $(n_y - k) \times (n_x - k)$ elements are zero by definition both for JSAEs and standard TopK SAEs. The histogram was collected over 10 million tokens from the validation subset of the C4 text dataset, which produced 10.24 billion feature pairs. (b) The cumulative distribution function of the absolute values of Jacobian elements, again demonstrating that JSAEs induce significantly more computational sparsity than traditional SAEs. Measured on layer 15 of Pythia-410m.

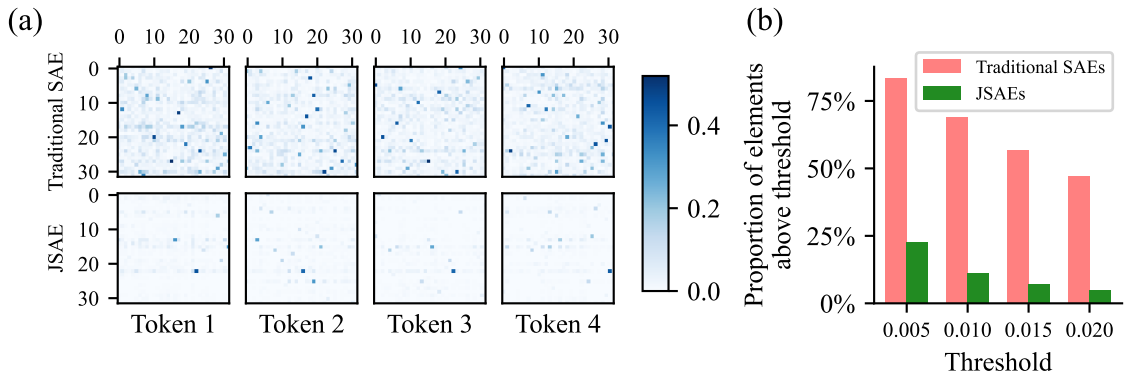
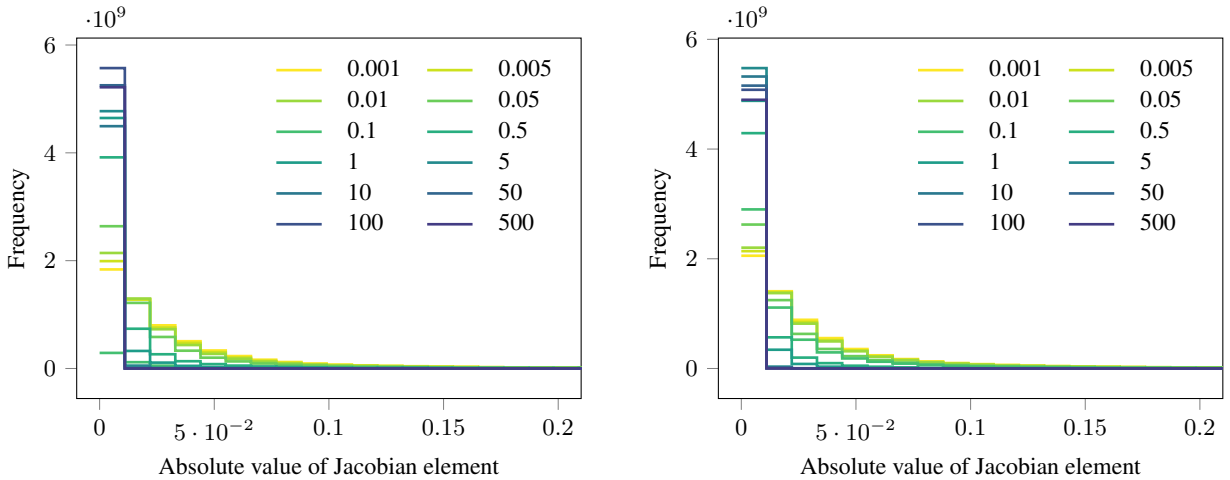


Figure 33. JSAEs induce a much greater degree of sparsity in the elements of the Jacobian than traditional SAEs. Identical to Figure 2 but measured on layer 3 of Pythia-70m.



(a) Pythia-70m Layer 3

(b) Pythia-160m Layer 7

Figure 34. Histograms that show the frequency of absolute values of non-zero Jacobian elements for different values of the coefficient of the Jacobian loss term. As the coefficient increases, the frequency of larger values decreases, i.e., the Jacobian becomes sparser. We provide further details in Figure 32.

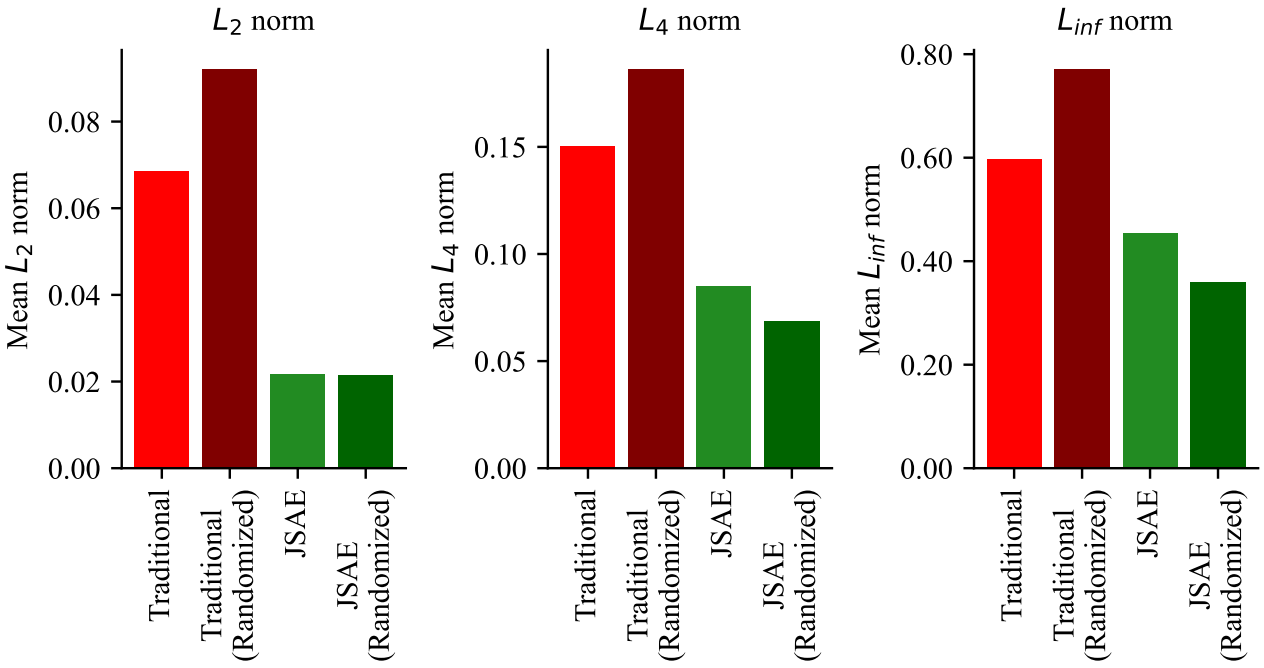


Figure 35. L_p norms of the Jacobians. We measure these by flattening the Jacobians and treating them as a vector. These results imply that the Jacobians are in fact becoming more sparse, as opposed to merely becoming smaller (see Section F.1). Averaged across 1 million tokens, measured on layer 3 of Pythia-70m.

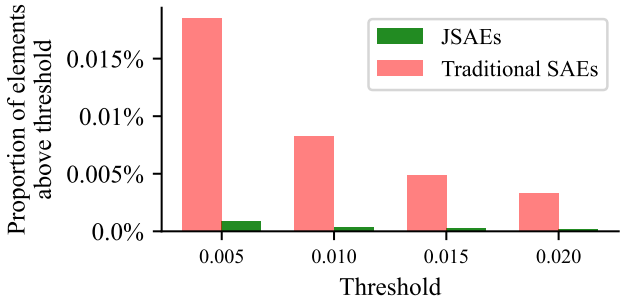


Figure 36. The Jacobians aren't only sparse locally (i.e. on each token in each prompt), but also globally (i.e. when averaged across many tokens), much more so than with traditional SAEs. In particular, here we consider the full $n_y \times n_x$ Jacobian (i.e. not slicing based on the TopK), which we average across 10 million tokens ($\frac{1}{N}\sum_{\text{prompt,token}} \mathbf{J}$) before considering its summary statistics. This is an important measure as it confirms that the connections found by JSAEs are indeed sparse in a global sense, not just when conditioning on a specific model input. Measured on layer 15 of Pythia-410m. Note that the small numbers on the y-axis are due to the fact that, unlike in e.g. Figure 2, here we set 100% to be $n_y \times n_x$ rather than $k \times k$. We also note that for each element in the Jacobian, we are only taking the average over the tokens on which the corresponding output SAE latent is selected by the TopK activation function (i.e. when at least one element in the row of the Jacobian is nonzero); this is important because otherwise this measure would significantly conflate the sparsity of the Jacobian itself with the sparsity of the activations of each individual latent.

Figure 37. Automatic interpretability scores of JSAEs are very similar to traditional SAEs. Measured on all layers of Pythia-70m using the "fuzzing" scorer from Paulo et al. (2024).

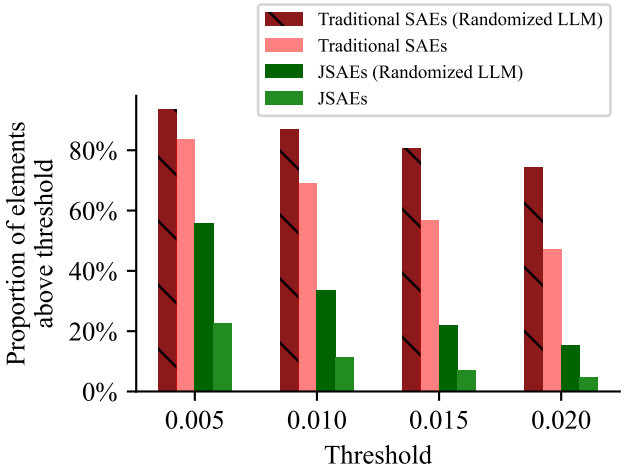


Figure 38. Jacobians are substantially more sparse in pre-trained LLMs than in randomly initialized transformers. This holds both when you actively optimize for Jacobian sparsity with JSAEs, and when you don't optimize for it and use traditional SAEs. The proportion of Jacobian elements with absolute values above certain thresholds. The figure shows the proportion of Jacobian elements with absolute values above certain thresholds. Identical to Figure 7 but measured on layer 3 of Pythia-70m.

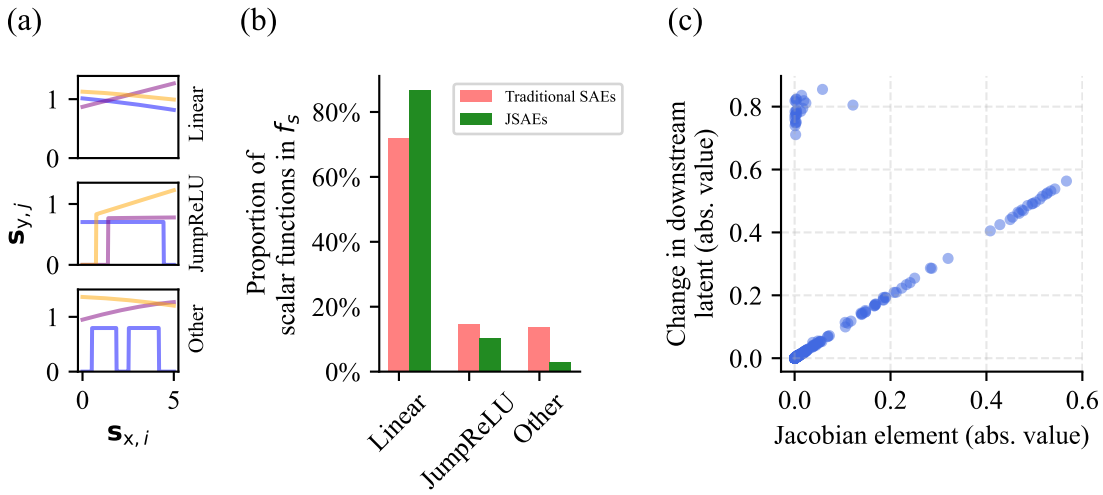


Figure 39. The function f_s, which combines the decoder of the first SAE, the MLP, and the encoder of the second SAE, is mostly linear. Identical to Figure 8 but measured on layer 3 of Pythia-70m.