A General Equilibrium Theory of Orchestrated AI Agent Systems

Version 2.1 (Corrected Price Simplex and Tâtonnement Normalization)

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Abstract

We establish a general equilibrium theory for systems of large language model (LLM) agents operating under centralized orchestration. The framework is a production economy in the sense of Arrow–Debreu, extended to an infinite-dimensional commodity space following Bewley. Each LLM agent is modeled as a firm whose production set is a subset of the Hilbert space $H = L^2([0,T], \mathbb{R}^R)$ of metric trajectories. The orchestrator is the consumer, choosing a routing policy over the agent DAG to maximize system welfare under a budget constraint evaluated at functional prices.

This corrected version makes one point explicit: in the finite-dimensional projected economy, the admissible price set is the positive simplex

$$\Delta_p^{AK-1} = \left\{ p \in \mathbb{R}_+^{AK} : \sum_{a=1}^{A} \sum_{k=1}^{K} p_{a,k} = 1 \right\},$$

and the price update in the tâtonnement is the Euclidean projection onto that simplex after positive truncation. It is *not* an L^2 renormalization onto a unit sphere. This clarification preserves the compact-convex geometry required by Brouwer's theorem and aligns the numerical implementation with the economic interpretation of prices as nonnegative shadow values.

We prove existence of equilibrium, a functional Walras law, Pareto optimality, decentralizability of Pareto optima, and uniqueness with geometric convergence under a contraction condition. The orchestration dynamics define a Walrasian tâtonnement on a compact convex state space. The framework also admits a DSGE interpretation in which SLO parameters act as policy rates.

Keywords: general equilibrium, production economy, orchestration, large language models, functional prices, Hilbert space, Walras law, Pareto optimality, Bewley, DSGE.
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Contents

1 Introduction

1.1 The problem

Modern AI deployments increasingly behave as organizations: collections of specialized agents—language models, retrieval modules, classifiers, validators, planners—composed in directed acyclic graphs and coordinated by a central orchestrator. At each stage, the orchestrator selects which agent to invoke, in what order, and with what priority, subject to quality, latency, cost, and load constraints.

This architecture is common in practice but under-theorized. Heuristic routing rules, hand-tuned weights, and ad hoc score aggregation do not answer the fundamental questions:

- Does an optimal orchestration exist?
- Is it stable?

- Is it unique?
- Do the coordination dynamics converge?
- Can the allocation be interpreted as an equilibrium?

This paper gives a mathematical answer by treating orchestrated AI agent systems as production economies.

1.2 The approach: an Arrow–Debreu–Bewley production economy

The central observation is structural: an orchestrated AI system can be modeled as a production economy. The commodity space is not finite-dimensional but the Hilbert space

$$H = L^2([0,T], \mathbb{R}^R),$$

whose elements are metric trajectories over a finite horizon. Each agent is a firm with a production set $Y_a \subset H$, determined by its frozen parameters and execution constraints. The orchestrator is the consumer, selecting a routing policy to maximize welfare subject to a budget constraint evaluated at functional prices.

The finite-dimensional approximation is obtained through an SFSL map

$$p_m : H \to \mathbb{R}^K,$$

which extracts K summary statistics from a metric trajectory. This produces a sequence of projected economies in finite-dimensional spaces, where existence can be proved by Brouwer. The limit economy in H then follows by a Bewley-type argument.

1.3 Contributions

This paper makes six contributions.

1. It models an orchestrated AI system as an Arrow–Debreu production economy in a Hilbert commodity space.
2. It proves a *functional Walras law*

   $$\sum_{a=1}^{A} \langle p_a, z_a(p) \rangle_H = 0,$$

   where $z_a(p)$ is agent a's functional excess demand.
3. It proves existence of equilibrium via finite-dimensional approximation and Brouwer's theorem.
4. It establishes the two welfare theorems.
5. It proves uniqueness and geometric convergence under a contraction condition on the orchestration operator.
6. It clarifies that the price space in the projected economy is a *simplex*, and that the corrected tâtonnement uses simplex projection rather than spherical normalization.

1.4 Relation to existing work

The existence proof follows the tradition of Arrow–Debreu in finite dimensions and Bewley for infinite-dimensional commodity spaces. The welfare theorems are classical in general equilibrium theory. The present contribution is to port that architecture into orchestrated AI systems and to identify the correct geometric structure of the projected price space.

2 The economy of orchestrated agents

2.1 Commodity space and price space

Let $R \geq 1$ denote the number of metric dimensions and $T > 0$ the observation horizon. The commodity space per agent is the separable Hilbert space

$$H = L^2([0, T], \mathbb{R}^R),$$

with inner product

$$\langle f, g \rangle_H = \sum_{r=1}^{R} \int_0^T f_r(t) g_r(t) \, dt.$$

Let $A \geq 1$ be the number of agents. The full commodity space is

$$X = H^A = \prod_{a=1}^{A} H,$$

with inner product

$$\langle x, y \rangle_X = \sum_{a=1}^{A} \langle x_a, y_a \rangle_H.$$

By the Riesz representation theorem, $H^* \simeq H$. Therefore the price space is also X. A price vector is

$$p = (p_1, \ldots, p_A) \in X,$$

where $p_a(t) \in \mathbb{R}^R$ is the instantaneous shadow-value vector for agent a at time t.

The value of a bundle $x \in X$ at prices $p \in X$ is

$$\langle p, x \rangle_X = \sum_{a=1}^{A} \langle p_a, x_a \rangle_H.$$

2.2 Firms: LLM agents as producers

Each agent $a \in \{1, \ldots, A\}$ is a firm with frozen technological endowment θ_a.

Definition 2.1 (Production set)**.** The production set of agent a is a set $Y_a \subset H$ representing all metric trajectories feasible under technology θ_a.

Assumption 2.2 (Production regularity)**.** For each agent a:

(Y1) Y_a is nonempty, closed, and convex.

(Y2) Y_a is bounded: there exists $R_a > 0$ such that $Y_a \subset B_H(0, R_a)$.

(Y3) $0 \in Y_a$.

Given prices $p_a \in H$, the profit of firm a is

$$\pi_a(p) = \sup_{y \in Y_a} \langle p_a, y \rangle_H,$$

and the supply correspondence is

$$\eta_a(p) = \arg\max_{y \in Y_a} \langle p_a, y \rangle_H.$$

2.3 The consumer: the orchestrating firm

The orchestrator is the sole consumer. It selects a routing policy over a DAG $G = (V, E)$ with $|V| = A$ agents. Let $\mathcal{P}(G)$ be the finite set of admissible paths through the DAG.

Definition 2.3 (Routing policy)**.** A routing policy is a probability vector

$$\alpha \in \Delta^{|\mathcal{P}(G)|-1}$$

over the set of admissible DAG paths.

The routing policy induces an expected demand trajectory $d_a(\alpha) \in H$ for each agent a. The orchestrator maximizes a welfare function $W(\alpha)$, which encodes quality, latency, cost, and regularization objectives.

Assumption 2.4 (Consumer regularity)**.** The welfare function W is continuous, strictly quasi-concave on the routing simplex, and locally non-satiated.

The budget constraint is

$$\sum_{a=1}^{A} \langle p_a, d_a(\alpha) \rangle_H \le \sum_{a=1}^{A} \pi_a(p).$$

Under local non-satiation, the budget binds at optimum.

2.4 Equilibrium

Definition 2.5 (Orchestrated general equilibrium)**.** An orchestrated general equilibrium is a triple

$$(p, y, \alpha) \in X \times \prod_{a=1}^{A} Y_a \times \Delta^{|\mathcal{P}(G)|-1}$$

such that:

(E1) For each a, $y_a \in \eta_a(p)$.

(E2) α maximizes W over the routing simplex subject to the budget constraint.

(E3) Markets clear:

$$d_a(\alpha) = y_a, \qquad a = 1, \dots, A.$$

3 Functional Walras law

Define the functional excess demand of agent a by

$$z_a(p) = d_a(\alpha(p)) - y_a(p) \in H,$$

and the aggregate excess demand by

$$z(p) = (z_1(p), \dots, z_A(p)) \in X.$$

Theorem 3.1 (Functional Walras law)**.** *Under the production and consumer regularity assumptions, for every admissible price vector $p \in X$,*

$$\sum_{a=1}^{A} \langle p_a, z_a(p) \rangle_H = 0.$$

Equivalently,

$$\langle p, z(p) \rangle_X = 0.$$

Proof. By local non-satiation, the consumer budget constraint binds:

$$\sum_{a=1}^{A} \langle p_a, d_a(\alpha) \rangle_H = \sum_{a=1}^{A} \pi_a(p).$$

By definition of profit,

$$\pi_a(p) = \langle p_a, y_a(p) \rangle_H.$$

Subtracting the two identities yields

$$\sum_{a=1}^{A} \langle p_a, d_a(\alpha) - y_a(p) \rangle_H = 0,$$

which is the claim. □

Remark 3.2. The functional Walras law is not imposed as an axiom; it is a theorem produced by budget saturation and profit maximization.

4 Existence of equilibrium

4.1 The SFSL approximation from H to V_K

To work in finite dimension, define a bounded linear SFSL operator

$$p_m : H \to \mathbb{R}^K$$

that extracts K summary statistics from a trajectory. Let

$$p_m^\dagger : \mathbb{R}^K \to H$$

be a reconstruction operator, and define the approximation subspace

$$V_K = \Im(p_m^\dagger) \subset H, \qquad \dim V_K = K.$$

Assumption 4.1 (SFSL completeness)**.** The family $(V_K)_{K \geq 1}$ is nested and satisfies

$$\overline{\bigcup_{K \geq 1} V_K} = H.$$

4.2 The projected economy E_K

Fix K. Let $Y_a^K = \mathrm{Proj}_{V_K}(Y_a) \subset V_K$. Since projection preserves nonemptiness, compactness, and convexity in finite dimension, each Y_a^K is a nonempty compact convex subset of V_K.

Choosing an orthonormal basis of V_K, we identify V_K with $\mathbb{R}^K$. Thus the projected production vector is

$$y = (y_1, \ldots, y_A) \in \prod_{a=1}^{A} Y_a^K \subset \mathbb{R}^{AK}.$$

4.3 State space and compactness

The crucial point of this corrected version is the geometry of the projected price space.

Definition 4.2 (Projected price simplex)**.** The projected price space is

$$\Delta_p^{AK-1} = \left\{ p \in \mathbb{R}_+^{AK} : \sum_{a=1}^{A} \sum_{k=1}^{K} p_{a,k} = 1 \right\}.$$

Remark 4.3. The admissible price set is a positive simplex, not a Euclidean sphere. The economic meaning is that projected prices are nonnegative normalized shadow weights over the AK projected commodities.

Remark 4.4 (Why L^2 normalization is inadmissible)**.** Replacing the simplex Δ_p^{AK-1} by the unit sphere $S^{AK-1} = \{p : \|p\|_2 = 1\}$ breaks the proof at three points:

(i) **Convexity.** S^{AK-1} is not convex; Brouwer's theorem requires a convex domain.

(ii) **Nonnegativity.** On the sphere, price components can be negative, violating the interpretation of prices as nonnegative shadow values.

(iii) **Heavy tails.** The ratio p_i/p_j of a uniform unit vector on S^{K-1} has Cauchy tails (Student-t with 1 degree of freedom), rendering the Walras identity numerically unstable in finite precision.

The simplex Δ_p^{AK-1} avoids all three issues.

Let the routing simplex be denoted by

$$\Delta_\alpha^{|\mathcal{P}(G)|-1}.$$

Define the full projected state space

$$\mathcal{K} = \left(\prod_{a=1}^{A} Y_a^K\right) \times \Delta_p^{AK-1} \times \Delta_\alpha^{|\mathcal{P}(G)|-1}.$$

Lemma 4.5. *The set $\mathcal{K}$ is nonempty, compact, and convex.*

Proof. Each Y_a^K is compact and convex in finite dimension. The price simplex Δ_p^{AK-1} is compact and convex. The routing simplex is compact and convex. The Cartesian product of finitely many compact convex sets is compact and convex. □

4.4 The corrected equilibrium map

Define three update operators.

Production update. For $y \in \prod_a Y_a^K$, let $d(y, p, \alpha)$ be projected demand. Then

$$K_1(y, p, \alpha) = \mathrm{Proj}_{Y^K}\Big(y + \rho\big(d(y, p, \alpha) - y\big)\Big), \qquad 0 < \rho < 1. \tag{1}$$

Price update. Let $z(y, p, \alpha) = d(y, p, \alpha) - y$ denote projected excess demand. Define componentwise positive truncation by $[v]_+ = \max(v, 0)$. Then the corrected price update is

$$K_2(y, p, \alpha) = \mathrm{Proj}_{\Delta_p^{AK-1}}\big([\,p + \eta z(y, p, \alpha)\,]_+\big), \qquad \eta > 0. \tag{2}$$

Remark 4.6 (Critical correction)**.** In (2), the operator $\mathrm{Proj}_{\Delta_p^{AK-1}}$ is the Euclidean projection onto the simplex. It is *not* an L^2 renormalization $p \mapsto p/\|p\|_2$. The latter would move prices onto a sphere, destroy convexity of the admissible price set, and break the compact-convex structure required by Brouwer's theorem.

Routing update. Let

$$s_a(p, y) = \langle p_a, y_a \rangle, \qquad a = 1, \dots, A,$$

collect revenues/scores into a vector $s(p, y) \in \mathbb{R}^A$. Let V be the Bellman-DP value operator on DAG paths. Then

$$K_3(y, p, \alpha) = \mathrm{softmax}_\beta(V\, s(p, y)), \qquad \beta > 0. \tag{3}$$

The full equilibrium map is

$$K = (K_1, K_2, K_3) : \mathcal{K} \to \mathcal{K}.$$

Lemma 4.7. *Under the standing assumptions, K is continuous and maps $\mathcal{K}$ into itself.*

Proof. The production update is continuous because Euclidean projection onto a nonempty compact convex set is continuous. The excess demand map is continuous by construction. Positive truncation is continuous. Euclidean projection onto the simplex is continuous. The Bellman value map and the softmax are continuous. Hence all three components are continuous and preserve their respective constraint sets. □

Theorem 4.8 (Existence in the projected economy)**.** *For every $K \geq 1$, the projected economy admits at least one equilibrium*

$$(p^K, y^K, \alpha^K) \in \mathcal{K}.$$

Proof. By the previous lemma, $K : \mathcal{K} \to \mathcal{K}$ is continuous and $\mathcal{K}$ is nonempty, compact, and convex. Brouwer's fixed-point theorem therefore implies the existence of a fixed point

$$K(p^K, y^K, \alpha^K) = (p^K, y^K, \alpha^K).$$

By construction, the fixed-point conditions are exactly projected firm optimality, consumer optimality, and market clearing. □

4.5 Extension to the full Hilbert space

Theorem 4.9 (Existence in H)**.** *Assume SFSL completeness. Then, as $K \to \infty$, the family of projected equilibria admits a subsequence converging weakly in*

$$H^A \times H^A \times \Delta_\alpha$$

to an equilibrium of the full economy.

Proof sketch. The projected price vectors live in a simplex and are uniformly bounded. The projected production vectors are uniformly bounded because each production set is bounded. Hence weak compactness applies. Standard Bewley-type passage to the limit yields an equilibrium in the full economy. □

5 Welfare theorems

5.1 First welfare theorem

Definition 5.1. A feasible allocation (y, α) Pareto-dominates $(\tilde{y}, \tilde{\alpha})$ if

$$W(\alpha) \geq W(\tilde{\alpha})$$

and all firms weakly prefer y to $\tilde{y}$ at equilibrium prices, with at least one strict inequality.

Theorem 5.2 (First welfare theorem)**.** *Every orchestrated general equilibrium is Pareto-optimal.*

Proof. If a feasible allocation strictly improved consumer welfare without worsening any firm's value, it would contradict consumer optimality and firm optimality at equilibrium. □

5.2 Second welfare theorem

Theorem 5.3 (Second welfare theorem)**.** *Every Pareto-optimal feasible allocation can be decentralized as an equilibrium for a suitable choice of functional prices, provided production sets are convex and welfare is quasi-concave.*

Proof sketch. Apply a supporting-hyperplane argument in the dual of the commodity space, using convexity of production possibilities and quasi-concavity of welfare. □

6 Uniqueness and convergence

6.1 The corrected orchestration dynamics

In the projected economy, define the discrete-time dynamics

$$y^{n+1} = K_1(y^n, p^n, \alpha^n), \tag{4}$$

$$p^{n+1} = K_2(y^n, p^n, \alpha^n), \tag{5}$$

$$\alpha^{n+1} = K_3(y^{n+1}, p^{n+1}, \alpha^n). \tag{6}$$

Remark 6.1. The corrected price dynamics remain Walrasian in spirit: prices react to excess demand, remain nonnegative, and are renormalized as budget shares over the projected commodity coordinates. The normalization is simplex projection, not spherical rescaling.

Remark 6.2 (Simultaneity)**.** Equations (4)–(6) must be evaluated as a *simultaneous* update: α^{n+1} uses p^{n+1} and y^{n+1}, not the previous iterates.

6.2 Contraction condition

Let $x = (y, p, \alpha) \in \mathcal{K}$, and write the full update as

$$x^{n+1} = K(x^n).$$

Theorem 6.3 (Uniqueness and geometric convergence)**.** *Suppose*

$$\|DK\|_{\mathrm{op}} < 1$$

on $\mathcal{K}$. Then:

1. *the equilibrium is unique;*
2. *for every initial condition $x^0 \in \mathcal{K}$, the sequence x^n converges geometrically to the unique equilibrium x^*:*
$$\|x^n - x^*\| \le \kappa^n \|x^0 - x^*\|, \qquad 0 < \kappa < 1.$$

Proof. Since $\mathcal{K}$ is complete and K is a contraction, the result follows directly from Banach's fixed-point theorem. □

6.3 A sufficient condition

Proposition 6.4. *A sufficient condition for contraction is*

$$(1 - \rho)\, \|A_K\|_{\mathrm{op}}\, \beta\, P < 1,$$

where:

- *ρ is the production update rate,*
- *A_K is the projected price mechanism,*
- *β is the routing temperature,*
- *P is the depth of the DAG.*

Proof sketch. The Jacobian of the full operator factors into a production block, a price block, and a routing block. The chain rule gives an upper bound by the product of their operator norms. □

Remark 6.5 (Auto-correction of β)**.** If the contraction condition is violated, reduce β until it is satisfied:

$$\beta \leftarrow \frac{0.9}{(1-\rho)\,\|A_K\|_{\mathrm{op}}\,P}.$$

This preserves convergence guarantees at the cost of a softer routing policy.

Remark 6.6 (Why simplex projection matters)**.** The contraction theorem is naturally stated on a compact convex state space. Replacing simplex projection by L^2 spherical normalization changes the target geometry from a simplex to a sphere, which is not convex. This does not merely change notation; it alters the fixed-point setting itself.

7 DSGE interpretation

7.1 DSGE embedding

Let

$$x_t = (y_t, p_t, \alpha_t) \in \mathcal{K},$$

and let ε_t denote exogenous disturbances. Then the orchestration economy induces a nonlinear state-space system

$$x_{t+1} = K(x_t, \varepsilon_t).$$

Linearizing around equilibrium x^* gives

$$x_{t+1} - x^* = DK(x^*)(x_t - x^*) + D_\varepsilon K(x^*)\varepsilon_t.$$

When $\rho(DK(x^*)) < 1$, the equilibrium is dynamically stable.

7.2 The Taylor rule of orchestration

SLO parameters can be interpreted as policy rates. Let λ_{lat} and λ_{qual} be welfare weights on latency and quality penalties. Then one may consider updates of the form

$$\lambda_{\mathrm{lat},t+1} = \lambda_{\mathrm{lat},t} + \phi_{\mathrm{lat}}(\mathrm{latency}_t - \mathrm{target}_{\mathrm{lat}}),$$

$$\lambda_{\mathrm{qual},t+1} = \lambda_{\mathrm{qual},t} + \phi_{\mathrm{qual}}(\mathrm{target}_{\mathrm{qual}} - \mathrm{quality}_t),$$

which shift equilibrium without directly controlling individual agents.

8 Consolidated statement

Theorem 8.1 (General equilibrium of orchestrated AI agent systems)**.** *Let $A \geq 1$ agents operate on a finite DAG under centralized orchestration. Assume:*

1. *regular production sets;*
2. *regular consumer welfare;*
3. *an SFSL approximation family with dense union in H;*
4. *strictly positive routing temperature.*

Then:

(i) *for every projected dimension K, the projected economy admits at least one equilibrium;*

(ii) *the functional Walras law holds;*

(iii) every equilibrium is Pareto-optimal;

(iv) every Pareto optimum is decentralizable;

(v) if the full projected operator is a contraction, the equilibrium is unique and the tâtonnement converges geometrically;

(vi) the correct projected price geometry is the simplex Δ_p^{AK-1}, and the correct price update is simplex projection after positive truncation.

9 Open questions and research program

Several open directions remain.

1. **Equilibrium multiplicity and selection.** Outside the contraction region, multiple equilibria may exist.
2. **Strategic agents.** If agents have endogenous objectives, the model must be extended to a hybrid general equilibrium/game-theoretic setting.
3. **Learning agents.** If agent capabilities change over time, the production sets become endogenous.
4. **Asymmetric information.** Noisy metric observation leads to a principal–agent problem.
5. **Dynamic DAGs.** Entry, exit, and rewiring of agents require a perturbation theory for equilibria under graph changes.
6. **Numerical geometry.** A systematic comparison between simplex projection and alternative normalization schemes would clarify the boundaries of economically meaningful implementations.

10 Conclusion

We have established a general equilibrium theory for orchestrated AI agent systems in a Hilbert commodity space. The fundamental objects of equilibrium theory carry over: firms, prices, profits, budget balance, market clearing, welfare optimality, and tâtonnement.

The main correction emphasized in this version is geometric and operational: projected prices belong to a positive simplex, not to a Euclidean sphere, and the tâtonnement updates prices by simplex projection after positive truncation. This correction preserves the compact-convex setting required by Brouwer and aligns the dynamics with the interpretation of prices as nonnegative shadow values over projected commodities.

The broader conclusion remains unchanged: orchestration is not merely a collection of engineering heuristics. It can be treated as an economy, and equilibrium theory provides a mathematically coherent language for its existence, stability, and governance.

A Simplex projection formula

For completeness, the Euclidean projection of a vector $v \in \mathbb{R}^n$ onto the simplex

$$\Delta^{n-1} = \{x \in \mathbb{R}^n_+ : \sum_{i=1}^n x_i = 1\}$$

is

$$\mathrm{Proj}_{\Delta^{n-1}}(v)_i = \max(v_i - \tau, 0),$$

where τ is chosen so that

$$\sum_{i=1}^{n} \max(v_i - \tau, 0) = 1.$$

Equivalently, if u is the decreasing rearrangement of v, then

$$\rho = \max\left\{ j \in \{1, \dots, n\} : u_j - \frac{1}{j}\left(\sum_{i=1}^{j} u_i - 1\right) > 0 \right\},$$

$$\tau = \frac{1}{\rho}\left(\sum_{i=1}^{\rho} u_i - 1\right).$$

This algorithm runs in $O(n \log n)$ and is due to Duchi et al. (2008).

B Why spherical normalization is not the right operator

Suppose one replaces the corrected price update

$$p^{n+1} = \mathrm{Proj}_{\Delta_p^{AK-1}}([p^n + \eta z^n]_+)$$

by

$$\tilde{p}^{n+1} = \frac{[p^n + \eta z^n]_+}{\|[p^n + \eta z^n]_+\|_2}.$$

Then:

1. the codomain becomes a sphere rather than a simplex;
2. positivity may be preserved, but the interpretation as normalized budget shares is lost;
3. convexity of the admissible price set is lost;
4. the fixed-point argument can no longer be phrased on the same compact convex state space.

Hence spherical renormalization is not merely a cosmetic alternative; it defines a different dynamical system.

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