

THE BELIEF–ACTION GAP IN LANGUAGE MODEL AGENTS

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ABSTRACT

Language model (LM) agents can fail for different reasons, requiring different explanations and interventions. We study how state beliefs, action values, and action preferences interact as LM agents reason and commit to decisions in text-rendered grid worlds. We estimate task-relevant beliefs using report-based and representation-based probes, and action values using inverse reinforcement learning. We find that task-relevant state and value information is often represented more reliably than it is reflected in behaviour. Tracking action preferences throughout reasoning further shows that decisions sharpen around a commitment point and reported belief uncertainty decreases near commitment, although individual belief changes do not consistently explain shifts in action preference. Finally, forced actions follow reasoning prefixes from other decisions when transplanted, whereas the tested reasoning-window activation edits provide limited control over the action. Together, these results suggest that some failures arise not from missing task-relevant information, but from how available information is translated into action. Our measurements distinguish information availability from belief-action consistency without by themselves identifying the cause of each failure. Our code is available at: <https://anonymous.4open.science/r/belief-action-gap-F745/>

1 INTRODUCTION

As language models are increasingly used as agents that observe, reason about, and act within external environments, evaluation based on outputs alone cannot diagnose why an agent fails (Wang et al., 2024). A suboptimal action may arise because the agent represents the current state incorrectly, because its preferences favour an undesirable successor state, or because available state and preference information fails to determine the selected action. These failure modes can produce similar behaviour while requiring different explanations and interventions (Amodei et al., 2016; Schmied et al., 2026).

We therefore analyse agent decisions in terms of three components: the agent’s *belief* about the current environment state, its *preference structure* over possible outcomes, and a *decision mechanism* that translates these beliefs and preferences into a selected action. This decomposition allows us to distinguish failures of state estimation from failures of preference, as well as cases in which relevant information is available but does not govern behaviour.

Existing work provides tools for studying parts of this process. Probing methods show that language model activations can encode spatial and other task-relevant information about the state of their environment (Li et al., 2022; Nanda et al., 2023; Gurnee & Tegmark, 2024; Arghal et al., 2026). Successful decoding, however, establishes information availability rather than functional use in behaviour (Hewitt & Liang, 2019; Belinkov, 2022; Orgad et al., 2025). Behavioural methods such as inverse reinforcement learning (Ng et al., 2000; Ziebart et al., 2008) can infer latent preferences from

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observed choices, while related work studies preferences in language-model agents through feedback and behavioural biases (Gao et al., 2024; Lu et al., 2025; Khan et al., 2026). Decision-theoretic work on the value of information formalises the cost of uncertainty for action selection (Howard, 1966). Chain-of-thought traces provide a useful view of decision formation but need not faithfully reflect the computations responsible for the final answer (Turpin et al., 2023; Lanham et al., 2023). Recent work also tracks how answer preferences, confidence, and uncertainty evolve during reasoning (Català et al., 2026; Hosseini et al., 2026; Caldarella et al., 2026), while other work decodes future responses from model internals (Belrose et al., 2023; Pal et al., 2023; Dong et al., 2025) and tests causal influence through activation interventions (Turner et al., 2023; Zhang & Nanda, 2024). Decodability or localisation also need not identify representations that provide behavioural control (Hase et al., 2023). Together, these methods do not by themselves explain where representation fails to translate into action.

We study how beliefs and preferences are transformed into action in a language model agent operating in a text-rendered grid world with known states, transitions, and optimal actions. We estimate task-relevant beliefs from internal activations and explicit reports, infer state preferences from behaviour using inverse reinforcement learning, track action preferences throughout reasoning, and intervene on both reasoning text and hidden representations. Together, these analyses test when task-relevant information is available, how action preferences form, and whether that information causally influences the selected action.

Our main contributions are:

- **Belief–action diagnostics.** We combine estimated state beliefs and inferred preferences to measure whether emitted actions are consistent with the information available to the agent.
- **Decision formation during reasoning.** We track evolving action preferences and identify a commitment point at which the preferred action stabilises, then examine how beliefs and hidden-state patterns change around this transition.
- **Causal tests of information use.** We show that reasoning-text interventions can strongly redirect action preferences, while targeted edits to highly decodable hidden representations are much less effective, separating information availability from causal control.
- **Empirical evidence for a belief–action gap.** We find that task-relevant information can remain available even when behaviour is suboptimal, and that improved action consistency after reasoning does not simply result from more precise state estimation.

2 ESTIMATING BELIEFS AND VALUES

An agent may fail by misidentifying its state, preferring the wrong outcome, or acting inconsistently with its available information. We separate these possibilities using an estimated state belief $\hat{q}_t(s)$, an inferred state-value function $g_\theta(s)$, and the emitted action a_t . Beliefs come from activation probes; values are inferred from observed choices using one-step inverse reinforcement learning. Together, they yield expected local regret, which we decompose into the belief–action gap and the expected value of perfect state information. We measure decoded belief uncertainty with Shannon entropy.

2.1 GRID WORLD ENVIRONMENT

We study a language model agent in a fully observable, text-rendered grid world, following the setup of Arghal et al. (2026). The grid contains walls, open cells, a goal cell, a locked door that blocks the route to the goal, and a key that the agent must collect before passing through the door. The key is collected automatically when the agent enters its cell, after which the door can be unlocked. Transitions are deterministic, with a valid movement changing the agent’s position by one cell, while an action directed towards a wall or locked door leaves the agent in its current position. Figure 1 shows an example grid. For a grid layout G , we represent the environment state at decision step t as $s_t = (x_t, y_t, h_t, d_t)$. (x_t, y_t) are the agent’s grid coordinates, $h_t \in \{0, 1\}$ indicates whether it is carrying the key, and $d_t \in \{0, 1\}$ indicates whether the door is open. The layout G specifies the fixed locations of walls, the key, the door, and the goal. For our analyses, we use a fixed 9×9 grid layout. The grid contains 39 walkable cells and admits 142 valid task states. At each step, the

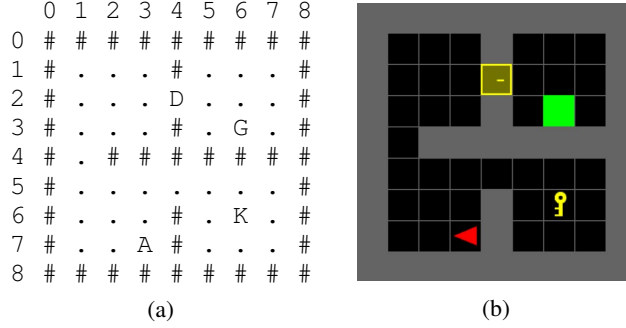


Figure 1: Example DoorKey environment shown in text and spatial form. The language model receives the text-rendered grid in (a), while (b) visualises the same underlying environment used to define states, transitions, and planner-optimal actions. A denotes the agent, K the key, D the door, G the goal, and filled cells denote walls.

language model agent receives the fully-observable grid, including coordinate labels, and its key-possession status, but no previous environment states or actions, making the interaction Markovian with respect to the task state. It then produces a sequence of reasoning tokens and emits one action from $\mathcal{A} = \{\text{LEFT}, \text{RIGHT}, \text{UP}, \text{DOWN}\}$.

2.2 ESTIMATING BELIEFS

We estimate the agent’s task-state beliefs using both representation-based and report-based estimates. Representation-based beliefs are obtained from probes trained on internal activations (Alain & Bengio, 2016; Gurnee & Tegmark, 2024). For a token region p and model layer ℓ , the probe input $\mathbf{u}_t^{(\ell,p)}$ consists of the activations from that region at decision step t . We train separate probes for the three components of the task state s_t : the agent position (x, y) , key possession h , and door status d . Position is treated as a single categorical variable over grid locations, while key possession and door status are binary. Their predicted distributions are combined into a factorised distribution over states and renormalised over the valid state set $\mathcal{S}_G$, giving the representation-based belief $\hat{q}_{t,p}(s)$.

We also obtain report-based estimates by directly querying the model about task-relevant features of the same environment state. We evaluate report accuracy using the highest-probability answer and measure uncertainty using Shannon entropy. Because these reports are elicited in separate forward passes, they are analysed separately and are not used to compute the belief–action gap. Full probe definitions and training details are provided in Appendix A.1, and the report-based protocol is described in Appendix A.2.

2.3 ESTIMATING VALUES

We infer the agent’s state preferences from observed state–action pairs using a one-step maximum-entropy inverse reinforcement learning model (Ng et al., 2000; Ziebart et al., 2008). Each state s is represented by an activation-derived feature vector $\phi_p(s)$ obtained by averaging the corresponding model representations across occurrences of that state. We fit separate models for the pre-reasoning and post-reasoning regions. The state-value function is linear in this representation: $g_{\theta_p}(s) = \theta^\top \phi_p(s)$. Since transitions are deterministic, an action is evaluated through the value of its successor state $T_G(s, a)$. We convert these successor values into action probabilities using a Boltzmann policy,

$$p_\theta(a \mid s, G) = \frac{\exp(\beta g_\theta(T_G(s, a)))}{\sum_{a' \in \mathcal{A}} \exp(\beta g_\theta(T_G(s, a')))}, \quad (1)$$

We then fit θ to the actions emitted by the language model by minimising their negative log-likelihood. We call this model one-step IRL as actions are compared through their immediate successor states rather than explicit trajectory rollouts. We interpret $g_\theta(\phi_p(s))$ as the agent’s inferred subjective value of state s . Further details on the state representations and model fitting are provided in Appendix A.3.

2.4 QUANTIFYING BELIEF-ACTION CONSISTENCY

We combine the estimated beliefs and values to characterise how the emitted action relates to agent’s inferred state belief and value function. For a known state s , we define the regret of action a as

$$\rho_\theta(s, a) = \max_{a' \in \mathcal{A}} g_\theta(T_G(s, a')) - g_\theta(T_G(s, a)). \quad (2)$$

The regret is zero when a leads to a maximally valued successor state and positive otherwise.

Because the agent may be uncertain about the current state, we first consider the expected local regret of the emitted action under its estimated belief,

$$\Delta_{t,p} = \mathbb{E}_{s \sim \hat{q}_{t,p}} [\rho_{\theta_p}(s, a_t)] = \mathbb{E}_{s \sim \hat{q}_{t,p}} \left[\max_{a' \in \mathcal{A}} g_{\theta_p}(T_G(s, a')) \right] - \mathbb{E}_{s \sim \hat{q}_{t,p}} [g_{\theta_p}(T_G(s, a_t))]. \quad (3)$$

This quantity compares the emitted action with the state-optimal action at each possible state and then averages the resulting regret under the estimated belief. We decompose it by adding and subtracting the value of the best single action under that belief: $\Delta_{t,p} = R_{t,p} + I_{t,p}$, where

$$R_{t,p} = \max_{a' \in \mathcal{A}} \mathbb{E}_{s \sim \hat{q}_{t,p}} [g_{\theta_p}(T_G(s, a'))] - \mathbb{E}_{s \sim \hat{q}_{t,p}} [g_{\theta_p}(T_G(s, a_t))], \quad (4)$$

and

$$I_{t,p} = \mathbb{E}_{s \sim \hat{q}_{t,p}} \left[\max_{a' \in \mathcal{A}} g_{\theta_p}(T_G(s, a')) \right] - \max_{a' \in \mathcal{A}} \mathbb{E}_{s \sim \hat{q}_{t,p}} [g_{\theta_p}(T_G(s, a'))]. \quad (5)$$

We refer to $R_{t,p}$ as the belief-action gap. It measures how much expected value is lost by the emitted action relative to the best single action under the agent’s estimated belief. A small value therefore indicates that the emitted action is consistent with the action preferred under that belief, whereas a large value indicates that another action has substantially higher expected value.

We interpret $I_{t,p}$ as the expected value of perfect state information. It measures the expected gain that would be available if the current state were known before choosing the action. This quantity is small when resolving the remaining state uncertainty would have little effect on achievable value and large when knowing the true state would substantially improve the action choice. We also quantify uncertainty in the decoded state belief using Shannon entropy: $H_{t,p} = -\mathbb{E}_{s \sim \hat{q}_{t,p}} [\log \hat{q}_{t,p}(s)]$. Unlike $I_{t,p}$, which depends jointly on the belief and inferred state values, $H_{t,p}$ depends only on the dispersion of the estimated belief. We test whether reasoning improves belief-action consistency and evaluate the changes in state uncertainty and its decision relevance, motivating our analysis of R , H , and I across reasoning.

3 LINKING BELIEFS AND VALUES TO BEHAVIOUR

We use grid world as a proof-of-concept setting in which states, transitions, and optimal actions are known exactly. With the belief and value machinery defined in the previous section, we ask whether suboptimal actions are better explained by inaccurate state beliefs, inappropriate state preferences, or a failure to translate available beliefs and preferences into action. We analyse 200 `gpt-oss-20b` grid-world trajectories comprising 3,194 decision steps. At each step, the model generates a reasoning trace and action using stochastic decoding at temperature 0.7 (Agarwal et al., 2025). For pre-reasoning, we force an action directly from the environment prompt before any reasoning tokens are generated.

3.1 INFERRED STATE BELIEFS AND PREFERENCES

We evaluate the state probes from before and after reasoning. Pre-reasoning representations use the three final prompt tokens, while post-reasoning representations use the three final reasoning tokens immediately before action generation. We use activations from layers 7, 15, and 23 and partition states into 80% training and 20% held-out sets, with no state appearing in both. Further probe-training details are provided in Appendix A.1, with expanded decoding results in Appendix C.1.

Held-out `has_key` and `door_open` accuracy decreases from 100.0% before reasoning to 89.5% and 88.6% after reasoning. Location decoding also becomes less precise, with top-1 accuracy

falling from 34.8% to 13.1% and accuracy within Manhattan distance one from 95.5% to 60.8%. Thus, although coarse location information remains recoverable, the decoded location representation becomes substantially less precise after reasoning.

We also fit the one-step agent-supervised IRL model from §2.3 to recover the model’s inferred state preferences. The resulting value model reaches held-out action-prediction accuracies of 29.2% before reasoning and 45.2% after reasoning. As a control, we also train the same model using planner-optimal actions, for which task-optimal action structure is substantially more recoverable. Full IRL results, including sensitivity to the inverse-temperature parameter β , are provided in Appendix C.3 and Appendix C.2; we use $\beta = 1$ throughout the main analysis. We additionally analyse report-based beliefs in Appendix B.1. In three illustrative wall-hit failures, the model correctly reports that its chosen direction is blocked while nevertheless selecting that action.

3.2 BELIEF–ACTION CONSISTENCY

We combine the representation-based beliefs and agent-supervised value models using the quantities defined in §2.4. We compute the expected local regret $\Delta_{t,p}$ and its decomposition into the belief–action gap $R_{t,p}$ and expected value of perfect state information $I_{t,p}$ before and after reasoning. We report results over all 3,194 decisions, the 2,465-step subset where every state probe is correct in both conditions, and the 762-step held-out probe set containing states unseen during training. As shown in Tab. 1, reasoning decreases the belief–action gap across all three subsets while increasing both the expected value of perfect state information and decoded-belief entropy.

Quantity	All Steps		All Probes Correct		Probe Held-out	
	Pre	Post	Pre	Post	Pre	Post
Expected local regret Δ	0.681	0.582	0.676	0.554	0.700	0.670
Belief–action gap R	0.467	0.268	0.514	0.288	0.316	0.199
Value of perfect state information I	0.214	0.314	0.163	0.266	0.384	0.472
Decoded Belief entropy H	1.494	2.085	1.106	1.811	2.788	3.020

Table 1: Mean expected local regret Δ , belief–action gap R , expected value of perfect state information I , and decoded belief entropy H before and after reasoning. The expected local regret decomposes exactly as $\Delta = R + I$.

The decomposition shows why the change in expected local regret alone understates the change in belief–action consistency. On the held-out set, Δ decreases only from 0.700 to 0.670 because the decrease in R is partly offset by the increase in I . After reasoning, the emitted action is therefore more consistent with the decoded belief, while the decoded state distribution is more diffuse and resolving its uncertainty is more valuable.

Fig. 2 illustrates the same qualitative behaviour for a single decision. Before reasoning, the decoded belief is sharply concentrated while the emitted action disagrees strongly with the inferred preference model. After reasoning, the belief becomes less concentrated and the emitted action is more closely aligned with the inferred preferences.

Together, these results show that reasoning can improve belief–action consistency without producing a sharper state estimate. The decrease in R indicates stronger alignment between the emitted action and the action preferred under the decoded belief, while the increases in I and H show that the decoded state distribution is more diffuse and that resolving this uncertainty would be more consequential for the inferred decision. The endpoint comparison, however, does not reveal when these changes arise during reasoning or whether they develop gradually or around a distinct transition.

4 DECISION FORMATION THROUGH REASONING

We examine how beliefs and action preferences evolve during reasoning using 95 trajectories comprising 1,276 decisions. At intermediate prefixes, we force a choice among the four actions and use the logits as the current action preference. Report-based beliefs are additionally available for 7,084 prefixes from 46 decisions across 31 trajectories. We use these measurements to study when action preferences stabilise and how this relates to beliefs and hidden representations.

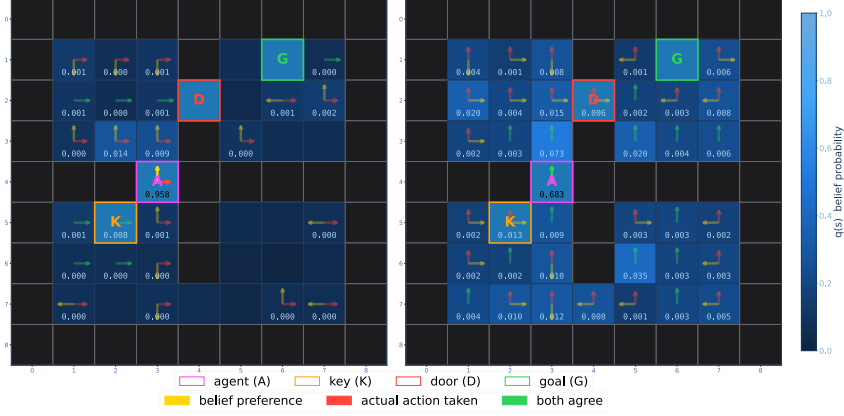


Figure 2: Example expected local regret under the pre-reasoning and post-reasoning conditions. Cell numbers show the probe-estimated location probabilities, with shading proportional to probability mass. Arrows mark the successor state preferred by the agent-supervised value model and the action actually emitted by the agent. In this example, reasoning increases the decoded belief entropy ($H : 0.261 \rightarrow 1.572$) while reducing the expected local regret ($\Delta : 2.386 \rightarrow 0.205$).

4.1 FORMATION OF ACTION COMMITMENT

At each evaluated prefix, we define the preferred action as the action with the highest forced-answer probability. We define *action commitment* as the earliest prefix after which this preferred action remains unchanged for the rest of the reasoning trace. Further details on prefix evaluation and commitment measurement are provided in Appendix D.1.

Action preferences become more concentrated on the action eventually emitted by the model. The mean probability assigned to the eventual action increases from 0.35 over the first tenth of the reasoning trace to 0.92 over the final quarter. Over the same interval, inverse normalised action entropy increases from 0.34 to 0.91. At the final evaluated prefix, the forced action agrees with the emitted action for all decisions (Fig. 3).

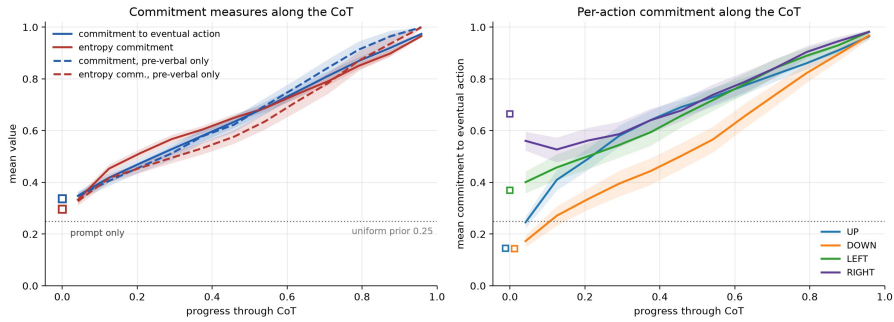


Figure 3: Evolution of action preference over reasoning (95 trajectories, 1,276 decisions, 98,310 evaluated prefixes). **left.** The mean probability assigned to the eventual action and inverse normalised action entropy as reasoning progresses, together with the same quantities before the first decisional action mention. **right.** The probability assigned to the eventual action separately by action. Bands are 95% trajectory-clustered bootstrap intervals.

This increase is not explained simply by the model repeating an action that it has already mentioned. Restricting the analysis to the 35,379 prefixes before the first decisional mention of an action produces nearly the same pattern, with eventual-action probability increasing from 0.35 to 0.92 and inverse normalised entropy from 0.33 to 0.89. Commitment also occurs substantially before reasoning

ends. The median commitment position is 0.49 of the reasoning trace, leaving a median 50.6% of the trace after the preferred action has stabilised. Among the 753 decisions for which an explicit textual commitment can also be identified, the forced-action preference stabilises before the textual commitment in 99.87% of cases, with a median lead of 523 reasoning tokens. This is consistent with recent findings that reasoning models can settle on an answer well before generation terminates (Boppana et al., 2026; Scalena et al., 2026).

Preference stabilisation is nevertheless gradual rather than a sharp transition, consistent with recent work studying premature confidence during reasoning (Gai et al., 2026). The probability that the preferred action changes at the next evaluated prefix falls from 0.133 in the first tenth of reasoning to 0.019 in the final tenth, and decisions undergo 6.0 preference switches on average before commitment. Across all evaluated prefixes, the preferred action belongs to the planner-optimal action set in 76.7% of cases. Changes in preference can therefore either improve or worsen the current decision, allowing us to ask whether such revisions are explained by corresponding changes in the model’s beliefs.

4.2 BELIEFS AROUND COMMITMENT

We next ask whether action commitment reflects a correction in the model’s reported understanding of the environment. We align report-based beliefs with the commitment point for the 45 decisions that contain reports both before and at or after commitment. Mean belief entropy decreases from 0.392 to 0.336 bits, a change of -0.056 with a 95% trajectory-bootstrap interval of $[-0.105, -0.009]$. Report accuracy, however, changes only from 82.8% to 83.0%, corresponding to an increase of 0.2 percentage points.

Individual changes in action preference are rarely accompanied by discrete corrections in the reported state. Across 682 action-change events, the correctness of at least one of the six current-state reports changes in only 14 cases, or 2.1%. All six reports remain correct in 66 of the 161 transitions from an optimal to a suboptimal preferred action. Thus, action stabilisation coincides with only modest changes in reported belief accuracy. The reports contain prospective information about future action degradation. Among prefixes whose current preferred action is optimal, a behavioural baseline predicts an optimality loss within the next three sentences with AUPRC 0.273. Adding report-based belief features raises AUPRC to 0.334, an improvement of 0.061 with a 95% trajectory-bootstrap interval of $[0.014, 0.123]$. Thus, report-based beliefs provide predictive information about future action degradation even though discrete belief corrections rarely coincide with individual action changes. Expanded transition-level results and uncertainty estimates are reported in Appendix D.3.

4.3 HIDDEN STATES AROUND DECISION CHANGES

We finally ask whether decision revision and commitment are reflected in the model’s hidden representations. We represent each reasoning sentence using mean-pooled activations and group these representations into activation-defined clusters. The clustering is unsupervised, with independent semantic annotations used only afterwards to aid interpretation. Full clustering methodology and comparisons are provided in Appendix D.4.

The distribution of these hidden states changes around commitment. Several clusters associated post hoc with solving and action consolidation become more common after commitment, whereas clusters associated with reading and reconstructing the environment become less frequent. The effect is not explained entirely by ordinary progression through the reasoning trace. For example, cluster C_{10} occurs in 14.0% of true commitment sentences compared with 5.9% of position-matched control sentences from the same decisions.

Related activation patterns also appear when the preferred action itself changes. Several clusters are significantly enriched at action-change events, and the same clusters are enriched when the preferred action changes from optimal to suboptimal and when it changes from suboptimal to optimal. These states therefore appear to reflect decision revision more generally rather than a representation specific to error detection or correction.

Taken together, the reasoning-time results show that action preference gradually sharpens and stabilises, typically well before reasoning ends. Reported belief uncertainty decreases modestly around commitment, but individual belief corrections account for few changes in action preference.

Hidden-state patterns nevertheless change around decision revision and stabilisation, suggesting that belief refinement and action commitment are related but distinct components of the decision process.

5 CAUSAL INFLUENCES ON ACTION SELECTION

The previous sections show what information is represented and when action preferences stabilise, but not which parts of this process causally influence the selected action. We therefore intervene on the reasoning text and on a reasoning-window activation site from which action decisiveness is highly decodable. Text interventions are stratified by the current probability of the eventual action, $c_{t,k} = \pi_{t,k}(a_t)$.

5.1 REASONING-TEXT PATCHING

We first test whether changing the reasoning trajectory changes the model’s current action preference. The experiment uses 396 mismatched prompt–reasoning pairs, combining the prompt from a decision A with a reasoning prefix from a different decision B whose emitted action differs from that of A . Prefixes are taken at approximately 10%, 50%, and 90% of B ’s reasoning trace, together with 396 congruent controls. Across all three cut positions, the forced-action distribution favours the action associated with the transplanted reasoning, with $\pi(a_B) \in [0.598, 0.673]$ compared with $\pi(a_A) \in [0.129, 0.165]$. Full results are given in Appendix E.1.

A second experiment tests whether a local textual intervention is sufficient to change the current action preference. It uses 240 reasoning prefixes across six bins of current eventual-action probability $c_{t,k} = \pi_{t,k}(a_t)$, with at most three prefixes from any one decision. At each prefix, a sentence asserting that the current runner-up action is correct is appended, together with neutral and supporting control sentences. Restricting to the 201 cases in which the injected runner-up differs from the action eventually emitted by the model, the forced top action switches to the injected action in 100% of cuts with $c_{t,k} < 0.7$, 92.5% for $0.7 \leq c_{t,k} < 0.85$, 90.0% for $0.85 \leq c_{t,k} < 0.95$, and 62.5% for $c_{t,k} \geq 0.95$. Susceptibility decreases with current action confidence (Spearman $\rho = -0.421$, $p = 5.0 \times 10^{-10}$) and remains significant after controlling for reasoning position (partial $\rho = -0.175$, $p = 0.013$). A neutral sentence changes the preferred action in only 9.6% of cuts, while a supporting sentence raises the mean probability of the current top action from 0.757 to 0.989. Reasoning text therefore has substantial causal influence on the current decision, although this influence weakens as the eventual action becomes more strongly preferred.

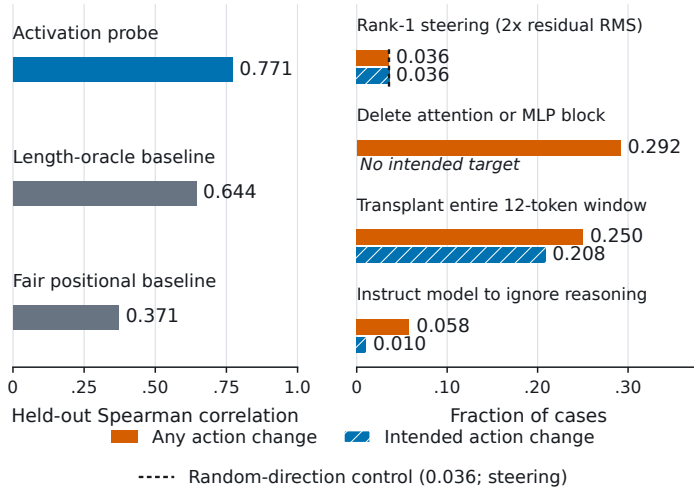


Figure 4: Decodability and intervention effectiveness at the reasoning-window site. **Left:** decodability of action decisiveness against positional and length-based baselines. **Right:** effectiveness of the tested activation interventions, distinguishing any change in the forced action from changes to the intended target action.

5.2 ACTIVATION PATCHING

We next test whether action choice can be controlled through a reasoning-window representation from which action decisiveness is strongly decodable. Following activation-patching and activation-steering approaches (Zhang & Nanda, 2024; Turner et al., 2023), we intervene on residual activations over a fixed window containing the final up to 12 reasoning tokens before the answer-forcing suffix. From this site, a non-linear readout predicts inverse normalised action entropy with held-out Spearman $\rho = 0.771 \pm 0.009$, compared with 0.644 for a length-oracle baseline and 0.371 for a positional baseline (Fig. 4, left).

Targeted edits at this activation site are largely ineffective (Fig. 4, right). Adding an action-associated rank-1 steering direction to the reasoning-window residual state (Turner et al., 2023) changes the action to the intended target in 3.6% of cases, matching the 3.6% rate for a norm-matched random direction. Replacing a low-dimensional subspace of the recipient representation with the corresponding donor representation produces no intended flips, while replacing the entire 12-token reasoning-window state redirects 20.8% of donor-recipient pairs to the donor-associated action at the best tested layer. In contrast to the strong effects of reasoning-text interventions, action decisiveness is readily decodable at this site, while the tested activation edits provide limited control over action identity. These results do not show that the represented information is causally unused, only that the tested interventions do not reliably control the action. This is consistent with prior work showing that localisation need not identify effective targets for behavioural editing (Hase et al., 2023). Full intervention sweeps and controls are reported in Appendix F.

6 DISCUSSION & LIMITATIONS

Our results suggest that suboptimal behaviour in language model agents cannot be explained solely by whether task-relevant state information is represented correctly. In §3, state information remains recoverable from model activations, yet the belief-action gap decreases after reasoning even as the decoded state distribution becomes less precise. Improved action consistency therefore does not appear to arise simply from forming a sharper estimate of the environment. The report-based analyses show that explicit state information does not always guide action. In three illustrative wall-hit cases, the model correctly reports that the chosen direction is blocked and still selects it. Around commitment, report accuracy changes little and belief corrections rarely coincide with action changes (§4.2). Action preferences sharpen gradually and usually stabilise around the middle of reasoning (§4). Before commitment, preferences may change repeatedly, but we find no consistent correction point linking belief updates to improved action choice. The causal interventions in §5 sharpen the distinction between information availability and causal control. Altering the reasoning text strongly changes the model’s current action preference, whether by replacing a reasoning prefix or injecting a single action suggestion. By contrast, targeted edits to a reasoning-window representation from which action decisiveness is highly decodable do not reliably steer behaviour under the interventions tested. Thus, decodability does not imply causal control (Belinkov, 2022; Hase et al., 2023). Our findings remain limited to a controlled deterministic grid world and to the estimators and interventions used here. Probe-based beliefs establish decodability rather than direct access to the model’s internal beliefs (Hewitt & Liang, 2019; Belinkov, 2022), and the one-step IRL model recovers an inferred preference structure rather than the model’s true value function. Likewise, the negative activation interventions apply only to the tested layers, subspaces, and edit families. Further work should trace whether the observed dissociation between representation, commitment, and behavioural control persists in richer environments and under stronger causal interventions.

7 CONCLUSION

We study how language model agents transform represented state information into action by jointly analysing beliefs, inferred preferences, reasoning dynamics, and causal interventions in a controlled grid world. We find that task-relevant information can remain available even when behaviour is suboptimal, and that the belief-action gap decreases after reasoning despite less precise decoded state beliefs. During reasoning, action preferences gradually sharpen and stabilise well before the trace ends, while changes in explicit belief correctness explain only a small fraction of action revisions. Causal interventions further show that reasoning-text edits strongly redirect action preferences,

whereas edits to a hidden representation predictive of action decisiveness are much less effective. Together, these results support treating state estimation, preference, and action selection as distinct sources of agent failure, and show that information being represented or decodable does not by itself imply that it governs behaviour.

AI USE STATEMENT

We used large language models to assist with code development and debugging, and editing parts of the manuscript. All AI-assisted code, and written content were reviewed by the authors before inclusion. Experimental results were obtained from the implemented analysis pipelines rather than generated by AI tools. We take responsibility for the final content of the paper, including all AI-assisted text, code, claims, and analyses.

ETHICS STATEMENT

This work studies decision making in language model agents using a controlled grid-world environment and does not involve human participants or personal data. Our experiments are designed to characterize how task-relevant information is represented, reported, and translated into actions, rather than to improve a model’s ability to manipulate or influence users. The intervention methods in this work modify reasoning text or internal activations to test their causal effect on action selection. Although we use these methods only as analysis tools in a simple research environment, related techniques could in principle be used to steer model behaviour in less controlled settings. Our results should therefore be interpreted as evidence about the mechanisms studied here. More broadly, we use terms such as belief, value, preference, and commitment as operational descriptions of measured model behaviour and internal representations. They should not be interpreted as evidence of human-like cognition, subjective experience, or intentionality.

REPRODUCIBILITY STATEMENT

We provide the experimental setup, state and belief definitions, value-estimation procedure, and main analysis methods in the paper. The appendix gives additional details on probe training, inverse reinforcement learning, reasoning segmentation, action commitment, report-based analyses, text interventions, activation interventions, controls, evaluation splits, hyperparameters, and robustness analyses. Code for reproducing the experiments is provided through the repository linked in the paper. The repository contains the analysis pipelines used to obtain the reported results. Analyses operate on externally supplied trajectories, activations, model responses, and intervention outcomes, with the required inputs specified for each command.

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A ADDITIONAL BELIEF AND VALUE ESTIMATION DETAILS

A.1 REPRESENTATION-BASED BELIEF ESTIMATION

For each decision step t , token region p , and model layer ℓ , we construct an activation representation $\mathbf{u}_t^{(\ell,p)}$ by concatenating the hidden states of the three tokens in the corresponding readout region. We consider the three final prompt tokens before reasoning and the three final reasoning tokens immediately before action generation. Each token activation has 2,880 dimensions, giving an 8,640-dimensional probe input.

We train separate probes for position, key possession, and door status. Position is represented as a categorical variable over walkable grid locations and is decoded using an MLP classifier. Key possession and door status are binary variables and are decoded using linear classifiers. All probes use the same state-disjoint split, with 80% of unique task states assigned to training and 20% to evaluation. All occurrences of a given state therefore appear in only one partition. Activation features are standardised using statistics computed from the training split.

Let $q_{t,p,\text{pos}}^{\text{probe}}(x, y)$, $q_{t,p,\text{key}}^{\text{probe}}(h)$, and $q_{t,p,\text{door}}^{\text{probe}}(d)$ denote the three predicted component distributions. We combine them into a factorised distribution over complete task states,

$$\tilde{q}_{t,p}(s) = q_{t,p,\text{pos}}^{\text{probe}}(x, y) q_{t,p,\text{key}}^{\text{probe}}(h) q_{t,p,\text{door}}^{\text{probe}}(d), \quad s = (x, y, h, d), \quad (6)$$

and renormalise over the valid state set $\mathcal{S}_G$,

$$\hat{q}_{t,p}(s) = \frac{\mathbb{1}[s \in \mathcal{S}_G] \tilde{q}_{t,p}(s)}{\sum_{s' \in \mathcal{S}_G} \tilde{q}_{t,p}(s')}. \quad (7)$$

Belief uncertainty is measured using Shannon entropy,

$$H(\hat{q}_{t,p}) = - \sum_{s \in \mathcal{S}_G} \hat{q}_{t,p}(s) \log \hat{q}_{t,p}(s). \quad (8)$$

Expanded held-out decoding results are reported in Appendix C.1.

A.2 REPORT-BASED BELIEF ESTIMATION

We obtain report-based beliefs by directly querying the model about features of the same rendered environment state supplied during the original decision. The full query registry contains 38 questions comprising 30 categorical questions and eight coordinate questions.

The categorical queries ask whether the model currently holds the key, whether the door is open, whether the cell immediately in each cardinal direction is a wall, whether the goal, key, or door is adjacent in each direction, and what would happen after each candidate movement. The latter queries test whether the movement would hit a wall, leave the agent holding the key, or leave the door open. Coordinate queries ask for the current locations of the agent, goal, key, and door and for the successor location under each candidate action.

Categorical prompts request one of $\{\text{yes}, \text{no}, \text{unknown}\}$. For repeated generations, the report distribution is the empirical frequency of each parsed answer. When candidate scores are available, we normalise their logits,

$$q_{t,p,j}^{\text{report}}(z) = \frac{\exp(\ell_{t,p,j}(z))}{\sum_{z' \in \mathcal{Z}_j} \exp(\ell_{t,p,j}(z'))}. \quad (9)$$

Responses that do not match the expected categorical or coordinate format are recorded as invalid rather than matched approximately. Report accuracy is computed from the most probable parsed answer, and uncertainty is measured using Shannon entropy.

Because report queries require a separate forward pass, they may elicit information that was not used in the original decision. We therefore analyse them separately rather than incorporating them into the belief-action gap.

Results from the failure-focused report analyses are provided in Appendix B.1.

We additionally examine a failure-focused set containing three decisions in which the emitted action points towards a blocked neighbouring cell. In all three cases, the model correctly reports that the selected direction is blocked while nevertheless choosing that action. These examples provide a direct case in which task-relevant state information is explicitly recoverable but does not determine behaviour.

A.3 ONE-STEP IRL ESTIMATION

For each reasoning position p , we represent an environment state s using the mean activation representation across occurrences of that state,

$$\phi_p(s) = \frac{1}{|\mathcal{I}_p(s)|} \sum_{t \in \mathcal{I}_p(s)} \mathbf{u}_t^{(p)}, \quad \mathcal{I}_p(s) = \{t : s_t = s\}. \quad (10)$$

Pre-reasoning representations are constructed from the prompt-suffix activations of the no-reasoning condition, while post-reasoning representations use the final reasoning-window activations. The representations are standardised separately before fitting the corresponding value models.

The value function is linear,

$$g_{\theta_p}(\phi_p(s)) = \theta_p^\top \phi_p(s), \quad (11)$$

and actions are evaluated through their deterministic successor states. The resulting Boltzmann policy is

$$p_{\theta_p}(a \mid s, G) = \frac{\exp[\beta g_{\theta_p}(\phi_p(T_G(s, a)))]}{\sum_{a' \in \mathcal{A}} \exp[\beta g_{\theta_p}(\phi_p(T_G(s, a')))]}. \quad (12)$$

Not every successor state encountered in a one-step comparison is necessarily represented in the trajectory data. We therefore retain a source-state observation only when activation-derived representations are available for the successors of all four candidate actions. This ensures that every training observation defines the complete four-action comparison appearing in Eq. (12).

For the agent-supervised model, the pre-reasoning value function is trained against the action produced directly from the environment prompt without reasoning, while the post-reasoning value function is trained against the action emitted after reasoning. This ensures that the representation and action labels correspond to the same decision condition.

We fit the models by minimising

$$\mathcal{L}(\theta_p) = - \sum_{(G, s, a) \in \mathcal{D}_p} \log p_{\theta_p}(a \mid s, G). \quad (13)$$

The optimisation uses Adam for 500 epochs with an initial learning rate of 10^{-3} , cosine learning-rate annealing, gradient clipping at norm 1.0, and L_2 regularisation. Source states are split into disjoint training and validation sets so that every occurrence of a source state belongs to only one partition. Unless otherwise stated, we use $\beta = 1$.

Robustness to the inverse-temperature parameter is reported in Appendix C.3.

A.4 PLANNER-SUPERVISED IRL CONTROL

As a control, we replace the agent’s realised action labels with planner-optimal actions while keeping the activation representations and value model otherwise unchanged. For each state s , the planner computes the set of shortest-path optimal actions

$$A_G^*(s) = \operatorname{argmin}_{a \in \mathcal{A}} D_G^*(T_G(s, a)). \quad (14)$$

A planner-optimal action is used as the supervision target during fitting, while evaluation can additionally treat any action in $A_G^*(s)$ as correct when multiple shortest-path actions are available.

This control asks whether the activation-derived state representations support a task-optimal action ordering more readily than they reproduce the agent’s realised policy.

B REPORT-BASED PROBE PROTOCOL

For report-based belief estimation, we use a total of 38 questions: 30 categorical questions and eight coordinate questions. Every prompt begins by describing the fully observable 9×9 DoorKey environment, giving the symbol legend, and defining RIGHT, LEFT, UP, and DOWN as cardinal directions in the rendered grid. It then supplies the current grid. Endpoint prompts include the carrying-key status for questions configured to require it; reasoning-time prompts include this status and the reasoning prefix $R_{t, \leq p}$.

Categorical question templates. The 30 categorical questions are generated from the following templates, where $d \in \{\text{RIGHT, LEFT, UP, DOWN}\}$, a ranges over the corresponding four moves, and $o \in \{\text{goal, key, door}\}$:

1. “Given the current grid only, do you currently have the key?” and “Given the current grid only, is any door currently open?” (two questions);
2. “Given the current grid only, is the cell immediately d of the agent a wall (#)?” (four questions);
3. “Given the current grid only, is the o immediately to your d ?” (12 questions);

4. “Given the current grid only, if you move a now, will you hit a wall?”, “will you have the key after that move?”, and “will the grid still show an open door (O) after that move?” (12 questions).

The requested output is exactly one label: A=yes, B=no, or C=unknown.

Coordinate question templates. Four questions ask for the current location of the agent, goal, key, or a door, and four ask for the agent’s location after each candidate move. The requested output is exactly `{"row": <int>, "col": <int>}`. For a current object that is not visible, the prompt specifies $(-1, -1)$; rows and columns are zero-indexed grid coordinates.

Post-processing and scores. Categorical text is lower-cased and stripped of surrounding quotation marks and terminal punctuation. Only an exact A/B/C or yes/no/unknown is accepted; other text is *invalid* rather than being matched fuzzily. For repeated generations, every sample, including *invalid*, contributes to the empirical distribution. For API log-probability runs, we exponentiate and renormalise the recognised A/B/C mass at the answer-token position and record missing candidates and excluded top- k mass. For the local scaled run, we score all three candidate tokens directly at the final answer position, divide the logits by temperature 0.7, and apply softmax.

For coordinate responses, we extract a JSON object and accept it only when both `row` and `col` are integers. Repeated coordinate samples are summarised by their modal valid coordinate; we additionally record parse rate, the number of distinct coordinates, and mean Manhattan error.

Panels used. The reasoning-time panel uses `wall_left/right/up/down`, `has_key`, and `door_open`, plus `hit_wall_after_a`, `has_key_after_a`, and `door_open_after_a` for the action read out at that position. Earlier small-scale reasoning runs additionally query `agent_location`, `goal_location`, `key_location`, and `door_location`; the scaled matched-cohort run defers these coordinate questions. Other endpoint analyses select question families from the same 38-question registry.

Query family	State-question pairs	Greedy	Ten-sample modal
Wall	132	93.9%	97.0%
Goal adjacency	132	100.0%	100.0%
Door adjacency	132	99.2%	100.0%
Key adjacency	132	67.4%	97.0%

Table 2: Report accuracy on the 33-state failure-mode set. Each family contains four directional questions per state. The modal result uses ten samples per question at temperature 0.7.

B.1 ADDITIONAL REPORT-BASED BELIEF RESULTS

We use the report-based estimates to test whether task-relevant information that can be explicitly recovered from the model is reflected in its behaviour. On the 33-state failure set, each directional query family contains 132 state-question pairs. Greedy accuracy is 93.9% for walls, 100.0% for goal adjacency, 99.2% for door adjacency, and 67.4% for key adjacency. Taking the modal answer from ten samples at temperature 0.7 increases these values to 97.0%, 100.0%, 100.0%, and 97.0%, respectively (Tab. 2).

We additionally analyse a 13-decision failure-focused slice containing four avoidable detours, three wall-hit decisions, two backtracks, and four optimal baseline decisions. A wall-hit decision is one in which the emitted action points towards a blocked neighbouring cell and therefore fails to move the agent in the intended direction. Reports are elicited from the same rendered environment while varying the fraction of the original reasoning trace shown to the model, without revealing the final action. In all three wall-hit failures, the model correctly reports that the direction ultimately selected by the agent is blocked. These examples illustrate cases in which the local information needed to avoid the error is explicitly recoverable despite the model selecting the blocked action.

Comparison with representation-based estimates. We also compare report-based and representation-based estimates on a matched set of 15 decision states. For each state, we ask

whether the adjacent cell in each cardinal direction is blocked. Report-based estimates identify the wall status correctly for every evaluated direction, whereas decoding from the original decision activations is less reliable (Tab. 3).

Direction	Report-based	Probe (Pre-reasoning)	Probe (Post-reasoning)
LEFT	100.0%	73.3%	73.3%
RIGHT	100.0%	86.7%	93.3%
UP	100.0%	100.0%	93.3%
DOWN	100.0%	80.0%	93.3%

Table 3: Accuracy of report-based and representation-based estimates of whether the adjacent cell in each direction is blocked, evaluated on the same 15 decision states.

The comparison should not be interpreted as showing that report-based beliefs are intrinsically more faithful than activation-based estimates. Reports are elicited in a separate forward pass and may recover information that was not used during the original decision. Rather, they show that relevant local state information can sometimes be explicitly retrieved even when it is not perfectly decoded from the original decision activations or reflected in the selected action.

C ADDITIONAL BELIEF AND VALUE RESULTS

C.1 EXPANDED STATE-DECODING RESULTS

The main text reports a compact set of metrics for representation-based state decoding. Here we provide the full held-out evaluation. Probe construction, training, and the state-disjoint split are described in Appendix A.1.

Key possession and door status are decoded reliably before reasoning and less accurately after reasoning. Exact position is substantially harder to recover. Across the 39 walkable grid locations, uniform top-1 chance accuracy is 2.56%. The position probe reaches 34.8% exact accuracy before reasoning and 13.1% after reasoning, remaining above this chance level in both conditions.

Although exact position decoding is limited, the decoded distribution is usually spatially close to the true location, particularly before reasoning. Before reasoning, the most probable decoded location is within Manhattan distance one of the true position in 95.5% of held-out examples and within distance two in 99.0%. The corresponding values after reasoning are 60.8% and 83.5%. The same pattern appears when considering the full probability distribution rather than only its most probable location.

Metric	Pre-reasoning	Post-reasoning
has_key accuracy	100.0%	89.5%
door_open accuracy	100.0%	88.6%
Location top-1 accuracy	34.8%	13.1%
Top-1 within Manhattan distance 1	95.5%	60.8%
Top-1 within Manhattan distance 2	99.0%	83.5%
Mean probability mass within distance 1	93.1%	59.2%
Mean probability mass within distance 2	97.2%	81.5%

Table 4: Expanded representation-based state decoding on held-out environment states. Location top-1 accuracy requires an exact match. “Top-1 within Manhattan distance k ” measures whether the most probable decoded location lies within distance k of the true location. “Mean probability mass within distance k ” is the probability assigned to all locations within that distance of the true position.

The decline after reasoning is therefore not restricted to exact location classification. Both local probability mass and binary state-variable decoding also become less precise at the post-reasoning readout.

C.2 PLANNER-SUPERVISED IRL CONTROL

The main analysis fits the one-step value model to the actions realised by the language model. As a control, we fit the same model using planner-optimal actions as supervision while leaving the activation-derived state representations and model class unchanged. For each source state s , the planner computes the set of shortest-path optimal actions $A_G^*(s)$. When several actions are tied, one is used as the training label, while evaluation can additionally count any member of $A_G^*(s)$ as optimal.

The purpose of this control is to distinguish two questions. Agent-supervised IRL asks how well the activation-derived state representation supports the model’s realised policy, whereas planner-supervised IRL asks how well the same representation supports the task-optimal action structure. The latter is substantially more recoverable, indicating that the state representations contain a cleaner task-optimal ordering than is expressed in the agent’s realised actions.

A separate four-room label-control experiment provides the same comparison while holding the activation inputs and training procedure fixed. Replacing agent-action labels with A^* -optimal labels increases test accuracy from 45.4% to 96.0%, and optimal-set membership accuracy from 36.9% to 99.2%. This separation therefore persists when the supervision labels are the main quantity changed.

Fig. 5 visualises the learned state values for planner-supervised and agent-supervised models for one task phase. The absolute magnitudes and signs should not be interpreted as distances to the goal. One-step IRL constrains values only through comparisons among successor states, so the meaningful quantity is the induced ordering of candidate actions rather than the absolute value assigned to an individual cell.

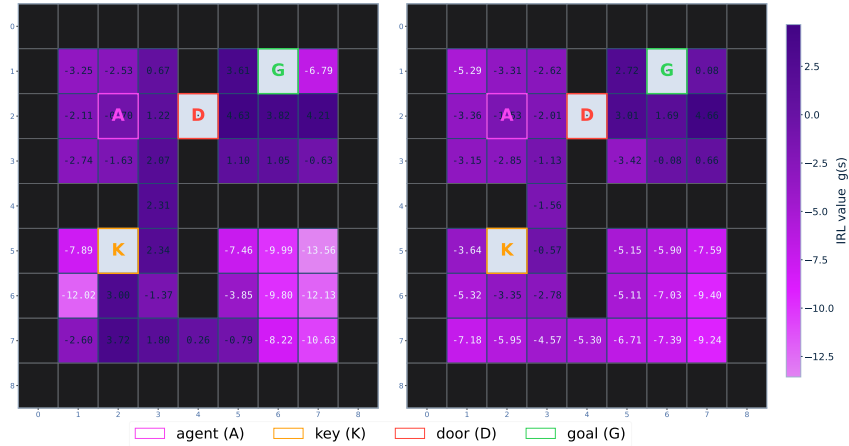


Figure 5: Example learned state values for planner-supervised IRL (left) and agent-supervised IRL (right), using post-reasoning activation-derived state representations. Darker cells indicate larger learned values. Because the one-step objective identifies relative successor-state preferences, absolute values should not be interpreted as goal distances; the comparison is intended to visualise differences in the learned state-preference structure.

C.3 AGENT-SUPERVISED IRL AND INVERSE-TEMPERATURE ROBUSTNESS

The agent-supervised value models are fitted separately for the pre-reasoning and post-reasoning conditions. The pre-reasoning model is supervised by the action elicited directly from the environment prompt without intervening reasoning, while the post-reasoning model is supervised by the action emitted after the reasoning trace. This aligns the action labels with the corresponding activation representations.

On held-out source states, the resulting model predicts 29.2% of pre-reasoning actions and 45.2% of post-reasoning actions. The lower pre-reasoning accuracy coincides with a substantially more imbalanced no-reasoning action distribution, making direct comparison of the two raw accuracies less informative than their use as condition-specific preference models in the belief–action gap analysis.

We additionally repeat the IRL and belief–action gap analyses across several values of the inverse-temperature parameter β . Changing β alters the sharpness of the induced Boltzmann policy and the numerical scale of the learned values, but does not change the qualitative endpoint result. Across the tested settings, the belief–action gap decreases from before to after reasoning while decoded belief entropy increases. We therefore use $\beta = 1$ for the main analysis.

D ADDITIONAL REASONING-TIME ANALYSES

D.1 REASONING SEGMENTATION AND COMMITMENT MEASUREMENT

For the reasoning-time analysis, we use 95 trajectories comprising 1,276 decisions and 2,370,850 reasoning tokens. The median reasoning length is 1,480 tokens per decision, with an interquartile range of 642–2,605 tokens and a maximum of 9,733.

We segment each reasoning trace into sentence-level units before aligning behavioural and activation measurements. A terminal JSON action of the form `{"action": "DIRECTION"}` is first removed, while embedded action JSON remains part of the reasoning trace. Sentence boundaries are introduced at blank lines and after periods, question marks, or exclamation marks followed by whitespace. Boundaries are not introduced after periods occurring in decimal numbers, coordinates, common abbreviations such as *e.g.*, *i.e.*, *etc.*, and *vs.*, or numbered action entries such as *1. LEFT*. Empty spans are discarded, and spans containing fewer than ten non-whitespace characters are merged with a neighbouring span.

We then tokenize the complete reasoning trace using the `gpt-oss-20b` tokenizer and assign each token to the sentence whose character span it overlaps. This yields 153,622 sentence-level reasoning units. The segmentation is fixed independently of the intermediate prefixes used for action elicitation.

For action-preference measurements, we evaluate at most 80 prefixes per decision. Consecutive prefixes are typically separated by 20 tokens, with wider spacing for longer reasoning traces, yielding 98,310 evaluated prefixes in total. At each prefix, we append a fixed answer-forcing suffix and obtain the logits for `{LEFT, RIGHT, UP, DOWN}`. Applying a softmax gives the prefix-level action distribution $\pi_{t,k}(a)$.

The preferred action at prefix k is

$$\hat{a}_{t,k} = \arg \max_{a \in \mathcal{A}} \pi_{t,k}(a). \quad (15)$$

We define action commitment as the earliest evaluated prefix after which this preferred action never changes again,

$$k_t^{\text{com}} = \min \{k : \hat{a}_{t,k'} = \hat{a}_{t,k} \quad \forall k' \geq k\}. \quad (16)$$

This is therefore the start of the final stable segment of the preferred-action sequence and is defined for all 1,276 decisions.

We additionally identify the first decisional mention of an action in the reasoning text. A decisional mention is an occurrence of UP, DOWN, LEFT, or RIGHT used as a candidate move, including bare direction words, mentions following a movement verb or answer cue, and capitalised answer-like uses. Descriptive uses such as “to the left of the door”, idioms such as “pick up the key”, and negated mentions such as “cannot go up” are excluded using fixed context rules.

Textual commitment is defined separately from forced-action commitment. It is the first sentence classified as concluding or answering that explicitly mentions the action eventually emitted by the model and is not followed by a sentence mentioning a different action. A textual commitment is identified in 753 of the 1,276 decisions.

D.2 ADDITIONAL COMMITMENT RESULTS

The prompt alone induces a non-uniform prior over actions. At the empty prefix, the mean probability assigned to the action eventually emitted is 0.34, compared with 0.25 under a uniform four-action distribution, and mean inverse normalised action entropy is 0.30. The initial preference is strongest for decisions that eventually emit RIGHT, where the mean initial probability is 0.66, compared with 0.37 for LEFT and 0.14 for both UP and DOWN.

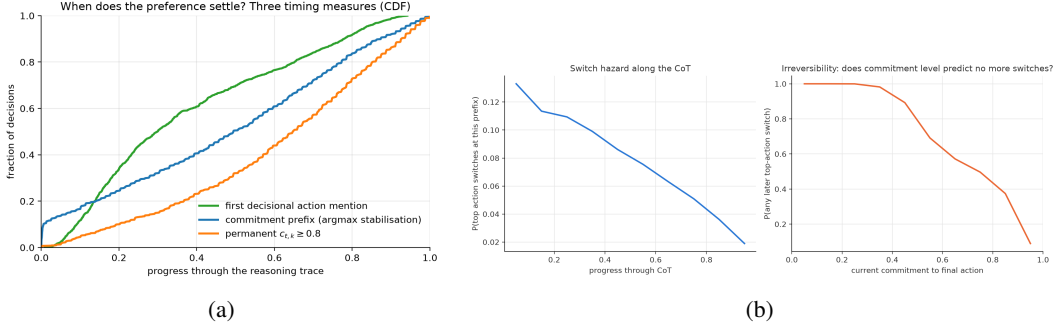


Figure 6: Additional commitment timing and stability results. **(a)** Cumulative distributions of the first decisional action mention, the action-commitment prefix, and the point after which the eventual-action probability remains above 0.8. **(b)** Left: probability that the preferred action changes at the next evaluated prefix as reasoning progresses. Right: probability of at least one later preference switch as a function of the current probability assigned to the eventual action.

To distinguish confidence in the eventual action from overall decisiveness, we track both $c_{t,k} = \pi_{t,k}(a_t)$ and inverse normalised entropy,

$$1 - \frac{H(\pi_{t,k})}{\log 4}. \quad (17)$$

Both increase throughout reasoning, as reported in the main text.

Commitment typically occurs around the middle of the reasoning trace. The median position of the first decisional action mention is 0.30 of the trace, the median action-commitment position is 0.49, and the median position after which $c_{t,k}$ permanently exceeds 0.8 is 0.65. Only 37.0% of decisions commit before the first decisional action mention.

A substantial portion of reasoning therefore follows action commitment. The median post-commitment share is 50.6% of the reasoning trace, corresponding to a median of 524 tokens and a mean of 887 tokens. The relative post-commitment segment is larger for shorter traces: median commitment position is 0.32 in the shortest quartile of reasoning length and 0.60 in the longest quartile. Because commitment is defined only over evaluated prefixes, the inferred commitment position is an upper-resolution estimate of the true stabilisation time.

We also examine how stable the current preference is before retrospective commitment. The probability that the preferred action changes at the next evaluated prefix falls steadily from 0.133 in the first decile of reasoning to 0.019 in the final decile. Decisions undergo 6.0 preference switches on average before commitment. Current confidence predicts, but does not guarantee, future stability: when $c_{t,k} > 0.9$, the probability of at least one later switch is still 0.089.

Commitment timing also differs across eventual actions. Decisions ending in RIGHT commit earliest, at a median normalised position of 0.39, whereas those ending in DOWN commit latest, at 0.61.

Reasoning content before and after commitment. For the 1,004 decisions with sentence-level function annotations, we classify 141,377 reasoning sentences into four coarse categories: world-reading, solving, meta-checking, and concluding or answering. Labels are produced by gpt-oss-20b using a fixed rubric adapted from Bogdan et al. (2025) and compared with 152 blind author-labelled sentences. Precision is 0.91 for concluding or answering, 0.78 for solving, 0.73 for world-reading, and 0.32 for meta-checking; we therefore treat the meta-checking category as exploratory.

Before commitment, 68% of sentences are classified as world-reading, compared with 29% after commitment. Solving increases from 25% to 45%, while concluding or answering increases from 1% to 12%. Against a placebo in which the commitment position is replaced by a position sampled from traces of similar length, the reduction in world-reading and increase in solving are larger than expected from ordinary progress through the trace, whereas the increase in concluding or answering is not.

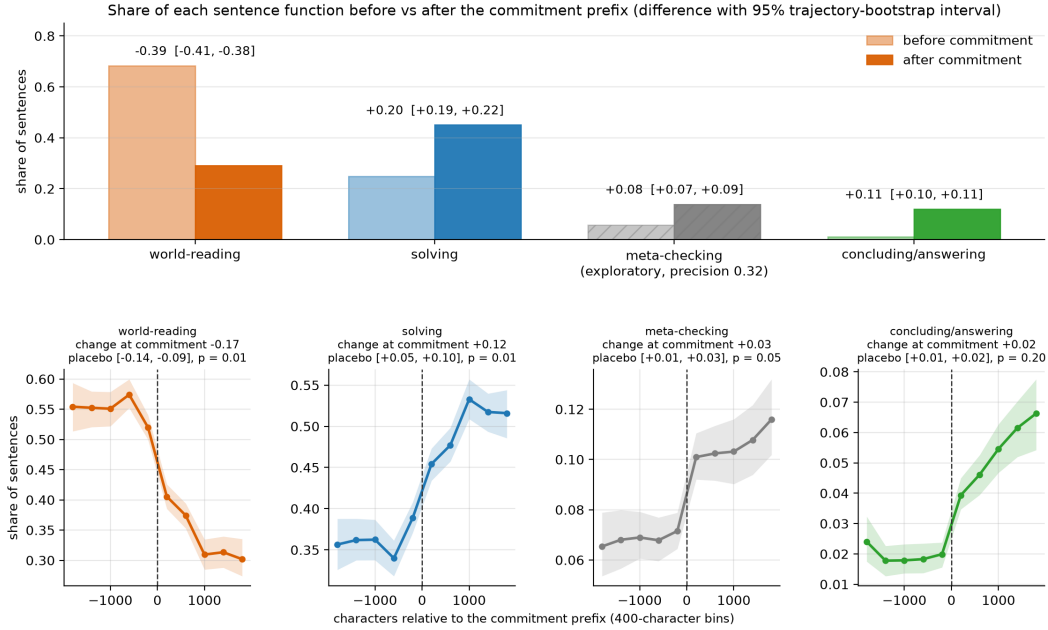


Figure 7: Sentence-function distributions around action commitment for 1,004 decisions and 141,377 sentences. The top panel compares function frequencies before and after commitment. The bottom panel shows their local evolution around the commitment position and compares the observed change with a position-matched placebo. Meta-checking should be interpreted as exploratory because its annotation precision is low.

Measure	Before	At or after	Change [95% CI]
Belief entropy (bits)	0.392	0.336	-0.056 [-0.105, -0.009]
Report accuracy	82.8%	83.0%	+0.2 [0.0, 0.6] pp

Table 5: Changes in report-based beliefs around retrospective action commitment. Values are averaged within decisions before being aggregated across decisions. Intervals are 95% trajectory-clustered bootstrap intervals.

A complementary keyword analysis gives the same broad pattern. After commitment, grid-related vocabulary falls to 0.37 times its pre-commitment density, coordinate pairs increase by a factor of 1.41, and checking-related vocabulary increases by a factor of 3.73. Direction words used as explicit answer mentions occur almost entirely after commitment, with 29 occurrences before and 1,751 after.

D.3 BELIEF CHANGES AROUND COMMITMENT AND ACTION REVISION

The reasoning-time report dataset contains 46 matched decisions from 31 trajectories, covering 7,084 sentence prefixes and 42,504 current-state reports. The reports comprise four local wall questions together with key possession and door status. Of these decisions, 45 from 30 trajectories contain reports both before and at or after the retrospective action-commitment point.

For each decision, we average report entropy and accuracy separately over prefixes before commitment and over prefixes at or after commitment, and then average these decision-level values across decisions. Mean report entropy decreases from 0.392 to 0.336 bits, corresponding to a change of -0.056 bits with a 95% trajectory-bootstrap interval of $[-0.105, -0.009]$. Mean report accuracy changes only from 82.8% to 83.0%, an increase of 0.2 percentage points.

We next test whether individual revisions of the preferred action coincide with discrete changes in report correctness. Across 682 action-change events, the correctness of at least one of the six

Transition	Events	Any report change	Δ error [95% CI]
Any action change	682	14 (2.1%)	0.0005 [−0.0022, 0.0027]
Optimal to suboptimal	161	6 (3.7%)	0.0000 [−0.0065, 0.0055]
Suboptimal to optimal	182	3 (1.6%)	−0.0018 [−0.0080, 0.0017]

Table 6: Changes in report correctness at action transitions. “Any report change” counts events where the correctness of at least one of the six current-state reports changes. Error changes compare the preceding sentence with the transition sentence.

current-state reports changes in only 14 cases, or 2.1%. For optimal-to-suboptimal changes, this occurs in 6 of 161 events, and for suboptimal-to-optimal changes in 3 of 182 events.

The corresponding change in mean report error is close to zero in all cases. For all action changes, the mean difference is 0.0005 with a 95% interval of [−0.0022, 0.0027]. For optimal-to-suboptimal changes it is 0.0000 with interval [−0.0065, 0.0055], and for suboptimal-to-optimal changes it is −0.0018 with interval [−0.0080, 0.0017].

Notably, all six current-state reports remain correct across 66 of the 161 optimal-to-suboptimal transitions. Thus, action quality can deteriorate even without a corresponding error in the reported local state.

Prospective monitoring. We additionally test whether current behavioural and belief readouts predict an optimality loss within the next three sentence boundaries. Among 5,283 prefixes at which the current preferred action is optimal, 347 are followed by such a loss, corresponding to a random-ranking AUPRC of 0.066. A behavioural baseline using reasoning progress, current action confidence, and the Jensen–Shannon divergence from the preceding action distribution reaches an AUPRC of 0.273 on held-out trajectories. Adding the observable belief reports raises AUPRC to 0.334, an improvement of 0.061 with a 95% trajectory-bootstrap interval of [0.014, 0.123]. The analogous next-sentence prediction is inconclusive.

Attention at action changes. As an exploratory analysis, we compare attention from the immediate action readout to the newest reasoning sentence at action-change events against a progress-matched stable-action control from the same decision. At layer 15, attention to the newest sentence increases by 0.0838 percentage points per token at action changes across 67 matched pairs, with a 95% interval of [0.0442, 0.1314]. Corresponding changes for the grid, key-status statement, and earlier reasoning include zero. This association is observational and does not establish that the attended sentence caused the action revision.

D.4 HIDDEN-STATE CLUSTERING ANALYSIS

We represent each reasoning sentence using mean-pooled layer-15 residual activations from `gpt-oss-20b`, yielding one 2,880-dimensional vector per sentence. For clustering, we sample 25,000 of the 153,622 sentence representations, reduce them to 100 principal components retaining 77.84% of the variance, and ℓ_2 -normalise the resulting vectors.

We use MiniBatch K-means with $K = 12$ clusters for the main analysis. Alternative clustering methods were also evaluated. HDBSCAN produces more locally separated clusters but labels 77.1% of sentence-mean representations as noise. Sentence-mean K-means provides complete coverage and stronger inter-cluster dispersion and Calinski–Harabasz score among methods that assign every sentence to a cluster. We therefore use K-means for the main reasoning-time analysis.

The clustering is entirely unsupervised. Semantic descriptions are added only after clustering by comparing assignments with independently generated reasoning-function annotations and by inspecting representative examples. Among 20,366 sentences with exact alignment to the semantic annotations, the normalised mutual information between K-means assignments and semantic labels is 0.131. We therefore use numeric cluster identities in the quantitative analysis rather than treating the clusters as semantic categories.

Cluster frequencies around commitment. The cluster distribution changes substantially after action commitment. C_{11} , C_2 , and C_7 increase in frequency by 12.8, 6.3, and 5.6 percentage points, respectively, while C_5 , C_6 , C_0 , and C_1 decrease by 9.8, 7.4, 4.4, and 2.7 percentage points. Post hoc semantic annotations associate the increasing clusters more strongly with solving and answering behaviour and the decreasing clusters more strongly with reading and reconstructing the environment.

To control for ordinary progress through the reasoning trace, we compare the cluster at each true commitment sentence with a position-matched sentence from the same decision. The largest difference occurs for C_{10} , which appears at 14.0% of commitment sentences compared with 5.9% of matched controls. C_9 and C_2 show smaller excesses of 3.2 and 2.5 percentage points.

Cluster	True commitment	Matched control	Excess
C_{10}	14.0%	5.9%	+8.2 pp
C_9	11.6%	8.4%	+3.2 pp
C_2	9.3%	6.8%	+2.5 pp
C_5	15.1%	13.5%	+1.6 pp
C_7	5.5%	4.5%	+1.0 pp

Table 7: Frequency of selected activation clusters at true commitment sentences and position-matched control sentences from the same decisions.

The clusters also differ in how action confidence evolves. C_{10} is associated with the largest average increase in probability assigned to the eventual action, 0.0245, together with an increase of 0.0400 in confidence and 0.0402 in action margin. In contrast, prefixes assigned to C_{11} already place a mean probability of 0.933 on the eventual action and show smaller subsequent changes.

Clusters at action revisions. Approximately 4.59% of reasoning sentences contain at least one change in preferred action. Relative to a within-decision shuffled-event control, C_6 , C_{10} , C_9 , and C_5 are enriched at these events by factors of 1.65, 1.64, 1.41, and 1.33, respectively. All four remain significant after false-discovery-rate correction with $q = 0.0024$.

The same clusters are enriched when action quality changes in either direction. For optimal-to-suboptimal transitions, the enrichment factors for C_6 , C_{10} , C_9 , and C_5 are 2.07, 1.48, 1.45, and 1.26. For suboptimal-to-optimal transitions, they are 1.77, 1.59, 1.44, and 1.35. This symmetry suggests that these representational states are associated with decision revision rather than specifically with detecting or correcting an error.

Cluster	Action change	Optimal to suboptimal	Suboptimal to optimal
C_6	1.65×	2.07×	1.77×
C_{10}	1.64×	1.48×	1.59×
C_9	1.41×	1.45×	1.44×
C_5	1.33×	1.26×	1.35×

Table 8: Enrichment of sentence-level activation clusters at changes in preferred action and preferred-action optimality relative to within-decision shuffled-event controls.

E ADDITIONAL REASONING-TEXT INTERVENTION RESULTS

E.1 WHOLE-CHAIN REASONING PATCHING

To test whether the reasoning trajectory itself influences the current action preference, we pair the prompt from a decision A with a reasoning prefix from a different decision B whose emitted action differs from that of A . We evaluate prefixes taken at approximately 10%, 50%, and 90% of B ’s reasoning trace. The analysis contains 396 mismatched prompt–reasoning pairs and 396 congruent controls, each evaluated in an independent forward pass.

Across all three cut positions, the forced-action distribution follows the action associated with the transplanted reasoning more strongly than the action associated with the original prompt. The

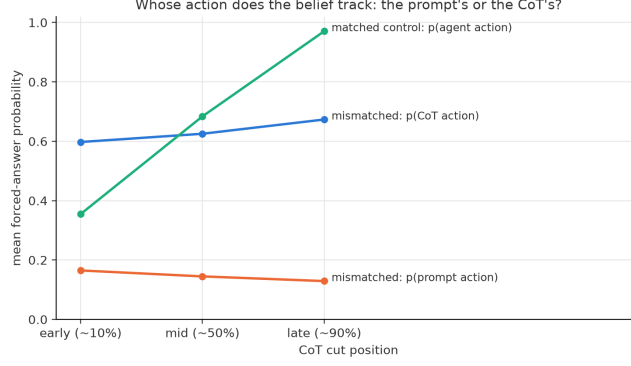


Figure 8: Whole-chain reasoning-text patching. The prompt from decision A is paired with a reasoning prefix from decision B , whose emitted action differs from that of A . Across early, middle, and late cuts, the forced-action distribution preferentially follows the transplanted reasoning rather than the original prompt. The experiment uses 396 mismatched pairs and 396 congruent controls.

probability assigned to a_B lies between 0.598 and 0.673, whereas the probability assigned to a_A lies between 0.129 and 0.165. At the late cut, the forced top action agrees with the reasoning-chain action in 98.5% of congruent pairs and 68.9% of mismatched pairs. The corresponding mean probabilities assigned to the reasoning-chain action are 0.971 and 0.673.

E.2 SINGLE-SENTENCE INTERVENTIONS

We next test whether a local textual intervention is sufficient to redirect the current action preference. We sample 240 reasoning prefixes across six bins of the current eventual-action probability $c_{t,k} = \pi_{t,k}(a_t)$, with 40 prefixes per bin and at most three prefixes from any one decision. At each prefix, we append one of three sentences:

1. a counter-evidence sentence asserting that the current runner-up action is correct;
2. a neutral sentence of similar form that names no action;
3. a supportive sentence asserting that the current top action is correct.

This yields 720 independent forced-action evaluations. The counter-evidence template is “Wait, I should double-check this. Looking at the grid again, the correct move here is ACTION.”

For the main analysis, we restrict to the 201 cases in which the injected runner-up action differs from the action eventually emitted by the model. The forced top action switches to the injected action in 100% of cuts with $c_{t,k} < 0.7$, 92.5% for $0.7 \leq c_{t,k} < 0.85$, 90.0% for $0.85 \leq c_{t,k} < 0.95$, and 62.5% for $c_{t,k} \geq 0.95$. Susceptibility decreases with current action confidence (Spearman $\rho = -0.421$, $p = 5.0 \times 10^{-10}$).

Action confidence is correlated with reasoning position ($\rho = 0.538$). The intervention effect also declines with progress through the trace ($\rho = -0.436$), but action confidence retains a smaller independent association after controlling for position (partial $\rho = -0.175$, $p = 0.013$).

The neutral sentence changes the preferred action in 9.6% of cuts overall, while the supportive sentence increases the mean probability assigned to the current top action from 0.757 to 0.989. These controls indicate that the counter-evidence effect is primarily driven by the semantic content of the injected sentence rather than by appending additional text.

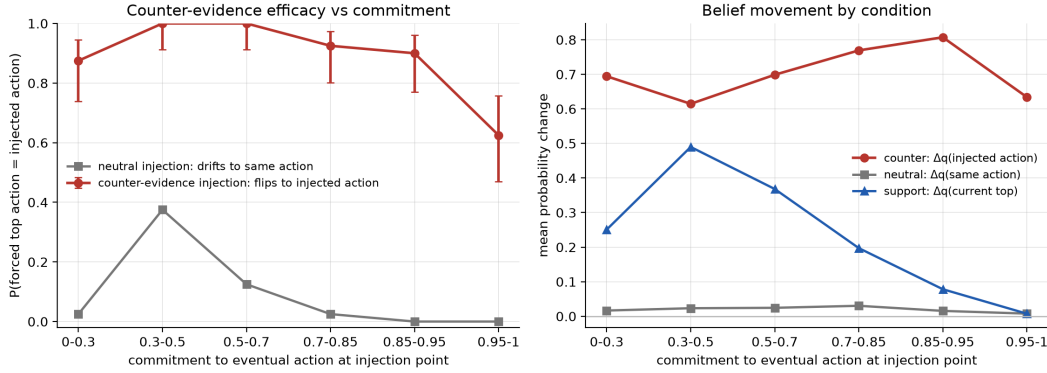


Figure 9: Single-sentence intervention results across 240 reasoning prefixes and 720 independent forced-action evaluations. **Left:** probability that the forced top action becomes the injected runner-up action as a function of current action confidence. **Right:** mean probability movement of the injected action under counter-evidence, neutral, and supportive interventions.

F ADDITIONAL ACTIVATION INTERVENTION RESULTS

F.1 COMMITMENT READOUT AND INTERVENTION SETUP

We target the residual activations over the final up-to-12 reasoning tokens before the answer-forcing suffix. A nonlinear readout trained on this reasoning-window representation predicts inverse normalised action entropy with held-out Spearman $\rho = 0.771 \pm 0.009$. For comparison, a length-oracle baseline reaches 0.644 and a positional baseline 0.371. Evaluation uses trajectory-grouped splits.

The interventions test whether the same representation can be manipulated to control the current action. We distinguish between any change in the forced top action and a change specifically to the intended target action. The main paper summarises these results in Fig. 4.

F.2 STEERING, TRANSPLANTATION, AND ABLATION CONTROLS

We evaluate several intervention families at the reasoning-window site.

Steering. We add an action-associated rank-1 direction to the reasoning-window residual state and compare it with a norm-matched random direction. At the representative magnitude shown in Fig. 4, both targeted and random steering change the action to the intended target in 3.6% of cases. The targeted direction therefore does not outperform the matched random control in this setting.

Transplantation. We replace components of the recipient reasoning-window representation with those from a donor prefix associated with a different preferred action. Low-rank transplantation does not produce intended flips in the tested settings. Replacing the entire 12-token reasoning-window state is more effective, changing the forced action in 25.0% of donor-recipient pairs and redirecting it to the intended donor-associated action in 20.8%.

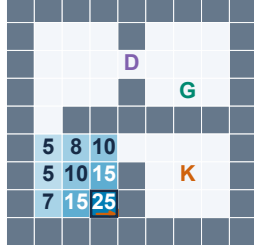
Sub-block deletion. Skipping an attention or MLP sub-block at one layer changes the forced action in up to 29.2% of cases. Because this intervention has no intended target, it is best interpreted as a disruption control rather than directed steering.

Instruction control. Appending an explicit instruction to disregard the mismatched reasoning changes the forced action in 5.8% of cases and moves it to the intended target in 1.0%.

These negative results should not be interpreted as showing that the reasoning-window representation is causally irrelevant. The tested interventions modify only particular layers, directions, and subspaces, and no known positive-control steering direction is available for this model. They do show, however, that the highly decodable representation at this site is not reliably writable under the interventions tested.

G ILLUSTRATIVE BELIEF-ACTION DECOMPOSITION

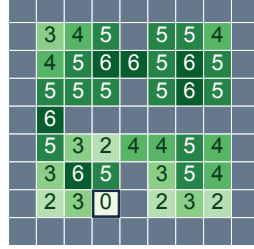
A A gridworld decision

Position belief $\hat{q}(s)$ 

Cell labels: probability (%)
K key D door G goal

$$Q(s, a) = g_\theta(T_G(s, a))$$

Illustrative state values



Outline: actual position
Same values across phases

Illustrative belief and state values

Expected action values

$$\mathbb{E}_{\hat{q}}[Q(s, a)]$$

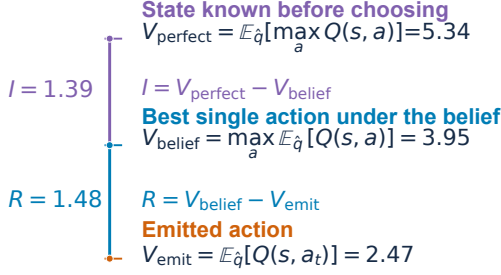
LEFT		3.49
RIGHT		2.47
UP		3.95
DOWN		2.12

UP: best under the belief

RIGHT: wall hit at actual state

All nine possible states: no key, door locked.

B Measure the belief-action gap

Probes $\rightarrow \hat{q}$ One-step IRL $\rightarrow g_\theta$ Expected local regret: $\Delta = R + I = 2.87$

C After reasoning

All 3,194 decisions

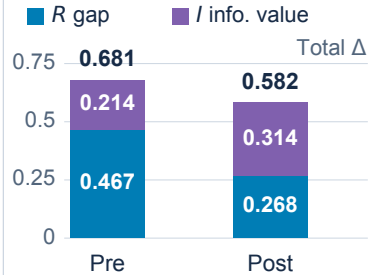
Decoded belief entropy
 $H: 1.494 \rightarrow 2.085$

Figure 10: **Belief-action decomposition: worked example and empirical summary.** (A–B) A constructed DoorKey example assigns belief probability to nine positions, all with no key and a locked door; blank belief cells have zero probability. The green map assigns values to all 39 non-wall positions. For this illustration only, $g(x, y, h, d) = v(x, y)$ across valid states; the door-cell value applies when the door is open. With $Q(s, a) = g(T_G(s, a))$, the emitted RIGHT action has expected value 2.47, the best single action UP has value 3.95, and choosing after learning the state yields 5.34. Thus the belief-action gap is $R = 1.48$, the value of perfect state information is $I = 1.39$, and expected local regret is $\Delta = R + I = 2.87$. RIGHT is blocked at the outlined actual position. These beliefs, values, and action are illustrative, not fitted observations or task rewards. (C) Empirical endpoint means across all 3,194 decisions (Tab. 1), using representation-based beliefs and agent-supervised value models fitted separately before and after reasoning. R decreases, while I and decoded-belief entropy H increase.