

MONTAGEFLOW: PHYSICALLY VALID HIGH FIDELITY EEG GENERATION

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ABSTRACT

Generative modeling offers a promising solution to the scarcity of labeled clinical electroencephalography (EEG) data under strict privacy constraints. Although recent generative models report substantial improvements in generation fidelity, we show that these gains conceal a categorical failure. EEG is not an arbitrary multichannel signal: each channel records the voltage difference between two scalp electrodes. So, real recordings satisfy exact linear identities among their channels. Current generators invariably violate these identities, yet this has gone unnoticed. We therefore propose **MontageFlow**, a flow-matching model that enforces these identities by restricting its output to signals that satisfy them. We find that ensuring validity improves fidelity too. Furthermore, to retain the EEG characteristics that oversmoothing washes out, we also condition the model on a physiological state vector encoded from the whole EEG window. For generation, this vector is drawn from a prior fitted in latent space. Across 3 large-scale benchmarks, our model improves TS-FID by 1.6–3.6× over the closest rival while ensuring validity.

1 INTRODUCTION

Clinical electroencephalography (EEG) data are difficult to collect at scale. Recordings with clinically meaningful labels require expert review, and privacy constraints limit data sharing. Generative models offer a way to relieve this bottleneck, synthesizing data where labeled recordings are scarce or cannot be shared. Yet EEG generation remains relatively underexplored despite recent advances in diffusion and flow-based modeling (Ho et al., 2020; Lipman et al., 2023; Peebles & Xie, 2023). The few existing approaches adapt generative adversarial networks (Hartmann et al., 2018), diffusion models (Shu et al., 2025) and flow matching models (Wang et al., 2026), typically treating EEG as a generic multichannel time series. This treatment overlooks a defining property of clinical EEG: its channels are structurally constrained by how they are recorded.

Clinical EEG is acquired from scalp electrodes, but read as potential differences between pairs of them. The chosen set of pairs, called a *montage*, links electrodes into chains. Summing the differences along a chain cancels the intermediate electrodes and leaves only the difference between its endpoints. Two chains with the same endpoints must therefore sum to the same value at every timepoint. The montage used across our benchmarks contains sixteen electrode pairs and two independent loops. These impose two independent linear constraints, restricting the sixteen channel values at each timepoint to a fourteen-dimensional subspace. Real recordings satisfy these identities exactly, but samples synthesized by current generators do not.

This failure has remained hidden because standard evaluation metrics do not explicitly measure montage violations. More importantly, improved distributional fidelity does not guarantee physical validity. We therefore enforce these identities by construction rather than by penalizing violations. Specifically, we parameterize the model output within the subspace permitted by the montage. Invalid signals are thus excluded without introducing an additional tuning hyperparameter.

Enforcing the montage identities guarantees physical validity, but does not by itself ensure fidelity to real EEG. Capturing dominant low-frequency activity while preserving low-energy transients and

Code is available at <https://github.com/sudarshanregmi/montageflow>.

heavy-tailed amplitudes remains challenging. JET (Wang et al., 2026) addresses waveform fidelity through explicitly specified auxiliary temporal, spectral, and statistical objectives. We instead use learned window-level conditioning to help preserve these characteristics. Specifically, we encode each multichannel EEG window into a single physiological state vector and condition the flow-matching decoder on it. This vector provides the decoder with window-specific context to guide generation. At sampling time, the vector is drawn from a prior fitted in the encoder’s latent space.

In summary, we propose **MontageFlow**, a flow-matching framework for physically valid, high-fidelity EEG generation that combines montage-constrained outputs with learned window-level conditioning. Across TUSZ, TUEV, and TUAB, MontageFlow improves TS-FID by 1.6–3.6 $\times$ over the strongest baseline while satisfying the montage identities. Our contributions are summarized below:

- **Physically valid EEG generation.** We show that state-of-the-art generators synthesize physically invalid EEG signals and introduce the *montage index* to measure this (0 = perfectly valid). MontageFlow restricts its output to the subspace of valid signals (Section 3).
- **Conditioning mitigates oversmoothing and validity improves fidelity.** We observe that latent conditioning mitigates oversmoothing of EEG. Additionally, we find that enforcing validity improves fidelity, lowering TS-FID on all six splits by 7 to 51 points (Section 6).
- **State-of-the-art results.** Across TUSZ, TUEV and TUAB, MontageFlow improves TS-FID by 1.6–3.6 $\times$ over the strongest baseline, with a montage index of exactly 0 (Section 5).

2 PRELIMINARIES

We first derive the exact linear constraints imposed by the bipolar EEG montage, then show that standard flow matching does not guarantee that generated outputs satisfy them.

Bipolar montage geometry. Let $X = [\mathbf{x}(\tau_1), \dots, \mathbf{x}(\tau_L)] \in \mathbb{R}^{C \times L}$ denote an EEG window with $C = 16$ channels and L physical time points. A bipolar montage represents channel i as the potential difference between an ordered pair of electrodes:

$$x_i(\tau) = v_{a_i}(\tau) - v_{b_i}(\tau), \quad i = 1, \dots, C. \quad (1)$$

As illustrated in Figure 1(a), summing channels along a directed path cancels the potentials at intermediate electrodes, leaving only the difference between the path endpoints. The montage contains two pairs of paths with common endpoints. On the left hemisphere, channels 1–4 and 9–12 both connect Fp1 to O1. On the right hemisphere, channels 5–8 and 13–16 both connect Fp2 to O2. Consequently, the following equalities hold at every physical time point τ :

$$x_1(\tau) + x_2(\tau) + x_3(\tau) + x_4(\tau) = x_9(\tau) + x_{10}(\tau) + x_{11}(\tau) + x_{12}(\tau), \quad (2)$$

$$x_5(\tau) + x_6(\tau) + x_7(\tau) + x_8(\tau) = x_{13}(\tau) + x_{14}(\tau) + x_{15}(\tau) + x_{16}(\tau). \quad (3)$$

Figure 1(b) verifies Eq. 2 on a representative TUSZ window.

Montage constraint. Let $\mathbf{1}_4$ and $\mathbf{0}_4$ denote four-dimensional vectors of ones and zeros, respectively. The two path equalities can be encoded by the constraint matrix

$$A = \begin{bmatrix} \mathbf{1}_4^\top & \mathbf{0}_4^\top & -\mathbf{1}_4^\top & \mathbf{0}_4^\top \\ \mathbf{0}_4^\top & \mathbf{1}_4^\top & \mathbf{0}_4^\top & -\mathbf{1}_4^\top \end{bmatrix} \in \mathbb{R}^{2 \times 16}, \quad (4)$$

where the four column groups correspond to channels 1–4, 5–8, 9–12, and 13–16, respectively. Thus, $A\mathbf{x}(\tau) = \mathbf{0}$ at every physical time point. Since A acts along the channel dimension, the corresponding window-level constraint is

$$AX = \mathbf{0}_{2 \times L}. \quad (5)$$

The set of montage-valid channel vectors is therefore $\ker A = \{\mathbf{u} \in \mathbb{R}^{16} : A\mathbf{u} = \mathbf{0}\}$. Since the two rows of A are linearly independent, $\text{rank}(A) = 2$, and $\ker A$ is a 14-dimensional subspace of $\mathbb{R}^{16}$.

Standard flow matching. Let $E \in \mathbb{R}^{C \times L}$ have i.i.d. standard Gaussian entries and $t \in [0, 1]$ denote flow time. Linear flow matching interpolates $X_t = tX + (1 - t)E$ and trains a velocity network $u_\theta(X_t, t)$ to regress the target velocity $X - E$. Generation starts from $\hat{X}_0 = \varepsilon \sim \mathcal{N}(0, I)$ and

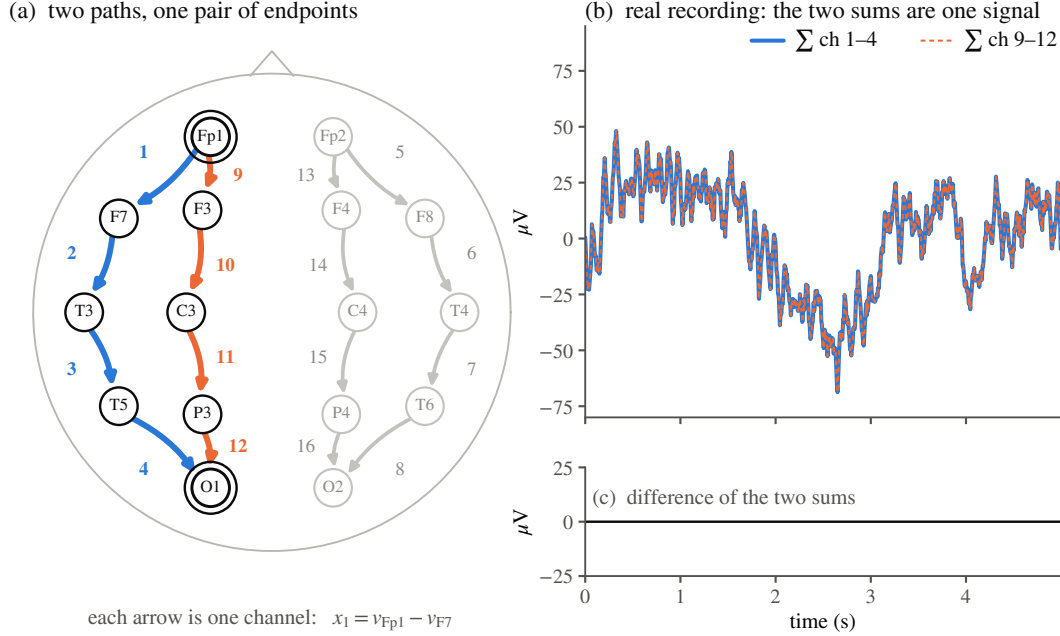


Figure 1: **The bipolar montage induces two exact linear constraints.** (a) Each channel is an oriented difference between two electrode potentials. Channels 1–4 and 9–12 form two paths from Fp1 to O1, and channels 5–8 and 13–16 do the same from Fp2 to O2. Either path sums to the potential difference between its endpoints. (b) The two left path sums overlap on a real TUSZ window, and (c) their difference is numerically zero.

integrates the learned velocity along its own trajectory $\hat{X}_t$: $\hat{X} = \varepsilon + \int_0^1 u_\theta(\hat{X}_t, t) dt$. Applying the montage constraint of Eq. 5 to this sample gives

$$A\hat{X} = A\varepsilon + \int_0^1 A u_\theta(\hat{X}_t, t) dt \quad (6)$$

where $A\varepsilon \neq 0$ almost surely and u_θ is unconstrained. Nothing forces the two terms to cancel, so standard flow matching does not guarantee montage-valid EEG.

3 MONTAGEFLOW

Overview. The overview of MontageFlow is shown in Figure 2. MontageFlow removes both sources of montage violation (Eq. 6) by construction: generation starts from noise projected onto $\ker A$, and the velocity network’s output is always projected onto $\ker A$. For fidelity, the velocity network is additionally conditioned on a global latent encoding of the target window.

Montage projector. Projecting the noise and the velocity onto $\ker A$ requires the same operation: given any channel vector $\mathbf{u} \in \mathbb{R}^{16}$, return a vector in $\ker A$. The vector $A\mathbf{u} \in \mathbb{R}^2$ holds the amounts by which $\mathbf{u}$ violates Eqs. 2 and 3 respectively. To make $\mathbf{u}$ valid, we cancel each violation by spreading it equally over the eight channels involved. Each of the four channels on the left-hand side of the equation is decreased by one eighth of the violation, and each of the four on the right-hand side is increased by the same amount:

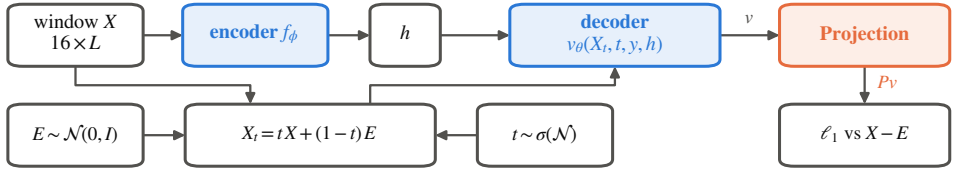
$$P\mathbf{u} = \mathbf{u} - \frac{1}{8}A^\top(A\mathbf{u}), \quad P = I - \frac{1}{8}A^\top A. \quad (7)$$

Since each row of A has eight entries equal to ± 1 and the two rows share no channel, $AA^\top = 8I_2$. Hence, $AP = A - \frac{1}{8}(AA^\top)A = 0$. Applied to a window X , P acts on each column, so $A(PX) = (AP)X = 0$. Therefore, P maps every window to a montage-valid window.

Projected velocity. With P in hand, the two projections of the overview are immediate. For the velocity, let $v_\theta(X_t, t, y, h) \in \mathbb{R}^{C \times L}$ be the raw output of the velocity network, conditioned on flow time t , class label y , and a latent h introduced below. MontageFlow uses the projected velocity

$$u_\theta(X_t, t, y, h) = P v_\theta(X_t, t, y, h), \quad (8)$$

1 Training



2 Generation

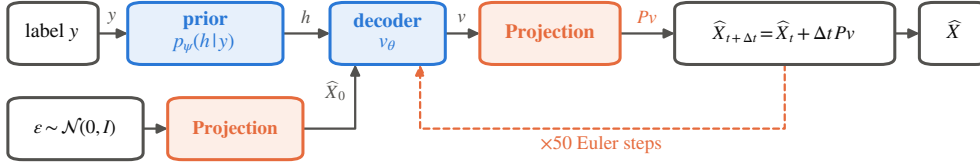


Figure 2: **Overview of MontageFlow.** During training, an encoder f_ϕ maps an EEG window X to a global latent h . The decoder receives the current flow state X_t , flow time t , class label y , and perturbed latent $\tilde{h}$, and predicts a velocity field. After encoder-decoder training, a class-conditional rectified-flow prior $p_\psi(h|y)$ is fitted to the encoded training latents and replaces the encoder during generation. The predicted velocity is projected onto $\ker A$. At inference, the Gaussian source is also projected to guarantee valid EEG.

which satisfies $Au_\theta = 0$. We sample $t = \text{sigmoid}(z)$, with $z \sim \mathcal{N}(-0.8, 0.8^2)$, and set $X_t = tX + (1-t)E$. We optimize the following ℓ_1 velocity-matching objective:

$$\mathcal{L}_{\text{MF}}(\theta, \phi) = \mathbb{E}_{X, y, E, t, \eta, \sigma} \left[\left\| (X - E) - P v_\theta(X_t, t, y, \tilde{h}) \right\|_1 \right]. \quad (9)$$

Here, $\tilde{h} = f_\phi(X) + \sigma\eta$ is the perturbed conditioning latent, defined in Eq. 12.

Validity by construction. The remaining source of violation is the start state, which we also project. Generation draws $\varepsilon \sim \mathcal{N}(0, I)$, starts from $\hat{X}_0 = P\varepsilon$, and integrates the projected velocity:

$$\hat{X} = P\varepsilon + \int_0^1 P v_\theta(\hat{X}_t, t, y, h) dt. \quad (10)$$

Applying A gives $A\hat{X} = AP\varepsilon + \int_0^1 AP v_\theta(\hat{X}_t, t, y, h) dt = 0$ since $AP = 0$. The generated sample now satisfies Eq. 5.

Latent conditioning. Clinical EEG varies widely from window to window in amplitude and rhythm. Generators trained with a regression loss can average this variety away and produce oversmoothed samples. To mitigate this, we give the velocity network a global view of the window it is generating by conditioning it on a latent physiological state vector:

$$h = f_\phi(X) \in \mathbb{R}^{256}, \quad (11)$$

computed by an encoder f_ϕ from the clean target window X during training. At generation time, the conditioning vector is sampled from a learned latent prior rather than obtained by encoding a real window. To make the decoder robust to deviations between prior samples and encoded latents, we perturb the latent during training:

$$\tilde{h} = f_\phi(X) + \sigma\eta, \quad \eta \sim \mathcal{N}(0, I_{256}), \quad \sigma \sim \mathcal{U}(0, 1). \quad (12)$$

The velocity network receives $\tilde{h}$ as conditioning in Eq. 9, and the encoder is trained jointly with it.

Latent prior. After jointly training the encoder and the velocity network, we replace the encoder with a learned prior. We encode the training set into a latent bank $\mathcal{H} = \{(f_\phi(X_i), y_i)\}_{i=1}^n$ and fit a class-conditional rectified-flow prior $p_\psi(h|y)$ to these pairs. Conditioning both the prior and the velocity network on y preserves the dependence between latent and label seen in training.

Sampling. To generate a window, we draw a class y , a latent $h \sim p_\psi(h|y)$, and a start state $\hat{X}_0 = P\varepsilon$, then integrate Eq. 10 with K Euler steps.

MontageDiff. Our method is objective-agnostic, so it also applies to denoising diffusion models (Ho et al., 2020). We term this variant MontageDiff (Appendix E).

4 EXPERIMENTAL SETUP

Datasets. We evaluate on the three Temple University Hospital (TUH) EEG corpora used by prior work: TUAB (abnormal/normal, 2 classes), TUEV (6 event types, heavily imbalanced), and TUSZ (seizure/background, 2 classes). All three are preprocessed with the released pipeline of Wang et al. (2026) into $16 \text{ channels} \times L$ length windows.

Table 1: Corpora used in our study. Windows lie on the 16-channel bipolar montage at 200 Hz.

	classes	window	train windows	test windows
TUSZ	2	16×1000	25,542	9,538
TUEV	6	16×1000	68,445	29,421
TUAB	2	16×2000	297,103	36,945

Metrics. Our primary fidelity metric is **(a) TS-FID** (lower is better): the Fréchet distance between representations of real and generated multivariate signals. It is computed from FFT magnitudes, so it sees spectral structure but is blind to physical validity. Hence, we propose the **(b) montage index**, which measures physical validity: 0 represents perfect validity, 1 is typical of independent channels. We also use **(c) shape correlation**, which compares the generated cross-channel correlation structure against the real one (higher is better, Appendix A).

Baselines. EEG-GAN (Hartmann et al., 2018), vanilla diffusion (Ho et al., 2020), vanilla flow matching (Lipman et al., 2023), JET (Wang et al., 2026), and its cross-channel variant JET[×].

Implementation. The encoder of Section 3 is a causal Transformer (8 blocks, width 512, 8 heads). The encoder output is projected to a single 256-dimensional latent h . The decoder is a 1-D DiT-B transformer (12 blocks, width 768, 12 heads) with adaLN conditioning on the timestep, the class label and h . Each channel is split into patches of size 100, and attention runs over the patches of all channels. Together they account for 156M parameters, of which the decoder holds 130M. Training runs for $\sim 160k$ steps at a batch of 128. We use AdamW with a learning rate of 5×10^{-5} held constant after 2.5 epochs of warmup. The prior is then fit on the encoded training split for 12k steps, and sampling (Figure 2) uses 50 Euler steps.

5 RESULTS

Table 2: Generation quality and physical validity (**bold**: best, underline: second best).

Method	TS-FID (train) ↓			TS-FID (test) ↓			montage index (test) ↓		
	TUSZ	TUEV	TUAB	TUSZ	TUEV	TUAB	TUSZ	TUEV	TUAB
EEG-GAN	342.3	316.9	394.5	331.4	400.9	455.7	0.539	0.519	0.505
Vanilla diffusion	113.2	82.4	120.1	121.6	145.8	149.1	<u>0.069</u>	<u>0.078</u>	<u>0.071</u>
Vanilla flow matching	125.0	123.9	157.1	137.0	212.8	186.0	0.283	0.437	0.294
JET	207.5	212.5	244.6	199.1	301.9	285.8	0.991	0.988	0.986
JET [×]	203.2	139.0	211.5	198.5	221.9	252.6	0.151	0.217	0.148
MontageDiff	25.2	27.9	31.1	40.6	89.6	50.2	0.000	0.000	0.000
MontageFlow	<u>31.5</u>	24.6	<u>41.0</u>	<u>44.0</u>	<u>93.1</u>	<u>59.9</u>	0.000	0.000	0.000

Generation quality and physical validity. Our models achieve the lowest TS-FID on both evaluation splits of all three datasets, and MontageFlow scores $1.6\text{--}3.6\times$ lower than the strongest baseline in each setting (Table 2). In every setting, the strongest baseline is a vanilla diffusion or flow-matching model sharing our backbone. Vanilla flow matching differs from MontageFlow only in latent conditioning and montage projection, providing a controlled comparison of their combined contribution. Their individual effects are examined in Table 5. Only real recordings, MontageFlow and MontageDiff achieve a montage index of exactly 0. JET has an index near 0.99, while adding cross-channel attention (JET[×]) reduces it to 0.15–0.22. Vanilla diffusion achieves the lowest index among the baselines, at 0.069–0.078, but still violates the montage identities.

Channel relationship. Montage validity alone does not guarantee realistic cross-channel relationships: any projected output satisfies the montage identities, even if its remaining channel dependencies differ from those of real EEG. We therefore evaluate shape correlation (Table 3). MontageFlow is closest to the real-data reference on all three corpora, achieving 0.955–0.990. The vanilla models, which also use cross-channel attention, achieve 0.93–0.96 but consistently remain below MontageFlow. JET scores near 0 across all settings, indicating poor agreement with the cross-channel structure captured by this metric. Adding cross-channel attention in JET[×] raises the scores to 0.91–0.95, supporting the importance of explicitly modeling channel dependencies. EEG-GAN achieves substantially lower scores of 0.41–0.67.

Downstream utility. We train the same classifier (CBraMod; Wang et al., 2025) for every method on 5,000 generated windows per class and test it on real windows (Table 4). We select the best epoch on the real validation split and average over three seeds. The grey row uses real windows. Our method improves both generative families. MontageFlow outperforms vanilla flow matching on all three corpora, and MontageDiff raises the average accuracy of vanilla diffusion from 0.632 to 0.716.

Table 3: Shape correlation on the test split ($\uparrow$, **bold**: best, underline: second best).

Method	TUSZ	TUEV	TUAB
EEG-GAN	0.412	0.674	0.668
Vanilla diffusion	0.928	0.948	0.961
Vanilla flow matching	0.926	0.935	0.963
JET	−0.033	0.215	0.024
JET [×]	0.916	0.908	0.948
MontageDiff	<u>0.948</u>	<u>0.973</u>	<u>0.981</u>
MontageFlow	0.955	0.977	0.990

Table 4: Downstream balanced accuracy ($\uparrow$, **bold**: best, underline: second best).

Method	TUSZ	TUEV	TUAB	Avg.
Real recordings	0.810	0.627	0.797	0.745
EEG-GAN	0.499	0.197	0.502	0.399
Vanilla diffusion	0.580	0.607	0.709	0.632
Vanilla flow matching	0.576	0.396	0.592	0.521
JET	<u>0.750</u>	0.401	0.581	0.577
JET [×]	0.663	0.404	0.623	0.563
MontageDiff	0.798	<u>0.589</u>	0.761	0.716
MontageFlow	0.604	0.570	<u>0.732</u>	<u>0.635</u>

6 ABLATIONS AND ANALYSES

Table 5: Incremental ablation from vanilla flow matching to MontageFlow (TS-FID $\downarrow$).

Method	TUSZ		TUEV		TUAB	
	train	test	train	test	train	test
Vanilla flow matching (x -parameterisation)	182.9	180.4	197.8	276.1	217.1	246.0
+ velocity parameterisation	136.9	138.2	150.7	233.9	181.5	215.2
+ cross-channel attention	125.0	137.0	123.9	212.8	157.1	186.0
+ latent conditioning	60.4	74.6	46.8	120.6	86.5	115.1
+ latent perturbation	49.8	60.8	31.8	106.3	81.7	110.5
+ montage projection	31.5	44.0	24.6	93.1	41.0	59.9

Incremental ablation with respect to vanilla flow matching. We present an incremental ablation in Table 5, which starts from flow matching and adds one component per row until it reaches MontageFlow. Switching from x - to velocity parameterisation lowers TS-FID by 31–47 on every split. Adding cross-channel attention gives the vanilla flow matching baseline of Table 2. It lowers TS-FID by 1–12 on TUSZ and by 21–29 on TUEV and TUAB. Latent conditioning (row 4) lowers TS-FID by a further 62–92, the largest gain in the ladder. Perturbing the latent during training (row 5) lowers it by another 11–15 on TUSZ and TUEV and by about 5 on TUAB. Finally, the projection buys physical validity outright for every generated EEG window. Notably, it also improves distributional fidelity, lowering TS-FID by a further 7–51 points on all six splits.

Table 6: **Is post-hoc projection enough?** TUAB test split (all corpora: Table 9). *Inference only* projects the model of row 5 of Table 5 at sampling time, and *training + inference* is MontageFlow (**bold**: best).

Projection	TS-FID $\downarrow$	PSD slope $\downarrow$	Envelope $\downarrow$	Hj. act. $\downarrow$	Hj. mob. $\downarrow$	Drift $\downarrow$
Inference only	140.6	0.629	0.244	0.256	0.366	0.097
Training + inference	59.9	0.400	0.164	0.215	0.232	0.092

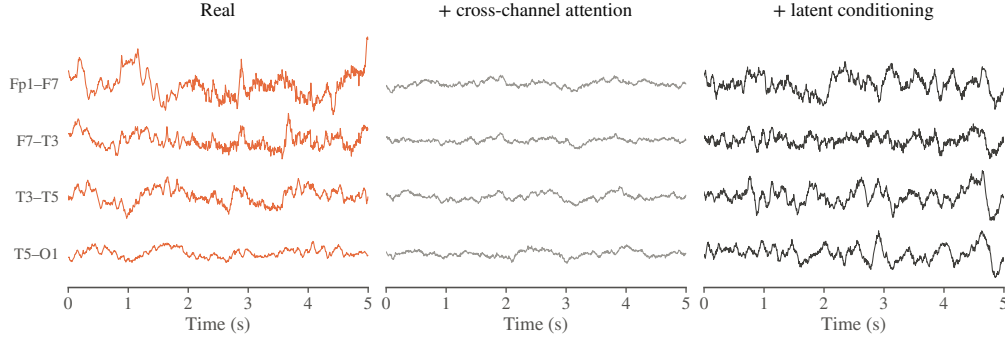


Figure 3: **Latent conditioning mitigates oversmoothing.** Representative samples from TUAB are shown.

Is post-hoc projection enough? The projection could instead be applied only at inference, as post-processing of the model trained without it (row 5 of Table 5). Both approaches make every generated window physically valid, but they differ sharply in fidelity. On TUAB, training with the projection outperforms post-hoc projection by 80.7 TS-FID points (59.9 vs. 140.6) and is also better on all five physiological metrics (Table 6). TUSZ and TUEV show the same trend (Appendix D). We hypothesize the unconstrained model unnecessarily spends capacity representing invalid signals.

Projected training noise. MontageFlow projects the initial noise at sampling time, but trains on unprojected Gaussian noise. We evaluate an alternative that also projects the training noise. This change worsens test TS-FID by 21.9 on TUSZ, 11.2 on TUEV, and 35.2 on TUAB. We hypothesize that unprojected noise adds regularization making the velocity network more robust.

Latent conditioning mitigates oversmoothing. Figure 3 shows four channels of real EEG (left column) and of generated EEG (middle and right columns). The two generated columns differ only in latent conditioning, isolating its effect on oversmoothing. Without the latent, generated windows collapse to a flat average and keep 42–52% of the real median amplitude. With it, they keep 78–84%.

Conditioning the prior on the label. The decoder is conditioned on both a window-level latent h and a class label y . Since the latent can encode class-related characteristics, sampling it from a marginal prior $p_\psi(h)$ independently of y can produce inconsistent latent-label pairs. We instead fit $p_\psi(h | y)$ and sample it with the same label that is passed to the decoder. On the test split, this improves TS-FID by 6.9 points on TUEV, which has 6 classes. The gain is trivial on TUSZ (1.1) and TUAB (1.5), which have 2 classes each.

The regression loss (L_1 versus L_2). Flow matching conventionally uses squared-error velocity regression. Compared with L_2 , L_1 places less emphasis on large residuals. We evaluate this choice by training MontageFlow under each loss. L_1 yields lower test TS-FID on all three datasets. Using L_2 instead raises it by 4.0 on TUSZ, 7.9 on TUEV, and 34.4 on TUAB.

Computational cost. We report the computational cost of every method on TUAB in Table 7. Vanilla diffusion and vanilla flow matching share the backbone and cost the same. MontageFlow and MontageDiff add an encoder with 26M parameters, which raises the time per training step by about 40%. At generation time, sampling the latent from the prior adds about 2–3 ms per window, so both generate at nearly the speed of the vanilla models (76 vs. 74 ms). JET and JET[×] are cheaper because they use a coarser patch, and EEG-GAN generates fastest but trains slowest.

Table 7: Computational cost across methods on TUAB.

Method	Patch	Params	s/step	GPU-h	ms/window
EEG-GAN	—	130M	2.40	36.0	2
Vanilla diffusion	100	130M	0.17	7.5	73
Vanilla flow matching	100	130M	0.17	7.5	74
JET	200	130M	0.11	5.0	40
JET [×]	200	130M	0.08	3.7	37
MontageDiff	100	156M	0.23	10.6	76
MontageFlow	100	156M	0.23	10.5	76

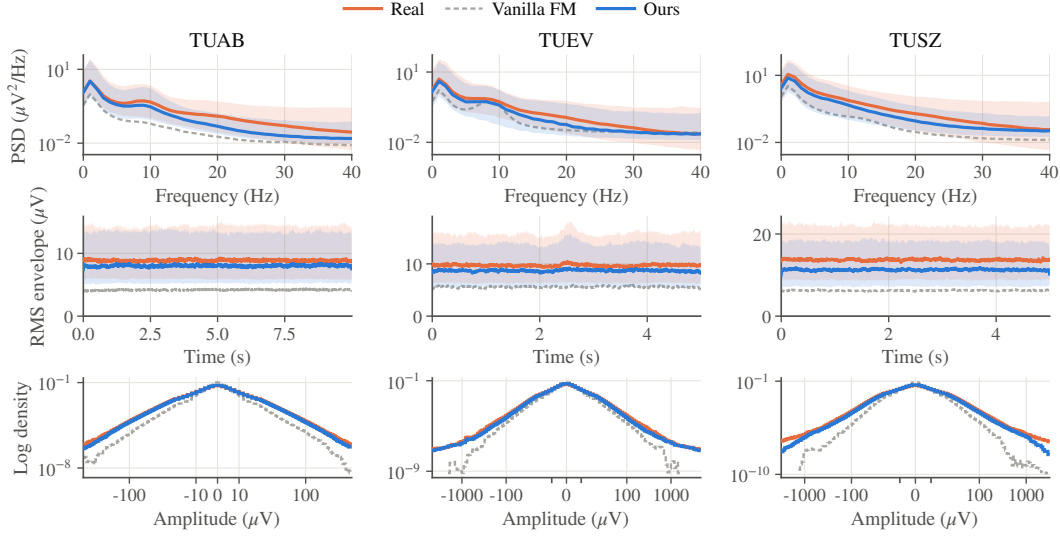


Figure 4: **Physiological diagnostics.** **Top:** power spectrum, median over windows, with shading from the 10th to the 90th percentile. **Middle:** amplitude over time (root-mean-square across channels), median over windows, with shading over the middle half of windows. **Bottom:** distribution of sample amplitudes (log density). Windows are 10 s on TUAB and 5 s on TUEV and TUSZ.

7 PHYSIOLOGICAL DIAGNOSTICS

We report five more diagnostic metrics in Table 8 (Appendix A). No method is closest to real EEG on every metric, but MontageFlow is closest on average, at 0.35 against 0.56 for the next best, MontageDiff. Furthermore, we qualitatively examine whether the generated signals behave like EEG by studying Figure 4. **(1) Power spectrum:** The top row shows the power at each frequency. Real EEG has most of its power at the lowest frequencies and falls off with frequency, with an alpha peak near 8–9 Hz on TUAB and TUEV. Below about 12 Hz, our spectrum follows the real shape and reproduces the alpha peak at the same frequency. Vanilla flow matching has less power at almost every frequency, and on TUEV it exaggerates the alpha peak. Above 12 Hz, our spectrum falls below the real one, most visibly on TUAB. However, real windows have much higher variance at these frequencies, and our median stays within the real shading throughout. **(2) Amplitude over time:** The middle row tracks signal amplitude across the window. Real EEG keeps a flat median envelope with no drift, and so does MontageFlow. Our envelope sits slightly below the real one, at 82–91% of its level, and its spread across windows is similar. Vanilla flow matching has about half the real amplitude. On TUEV, the real envelope rises near 2.5 s. MontageFlow reproduces this rise, while vanilla flow matching barely shows it. **(3) Heavy-tailed distributions:** The amplitude distribution of clinical EEG is heavy-tailed. A generator that collapses towards average behaviour loses the extremes, where transient events such as spikes live (bottom row). Our density follows the real curve from the sharp central peak down through the tails. Vanilla flow matching is too narrow everywhere, and its tails vanish by 1000 μV . Above 100 μV , our samples keep 60–70% of the real probability mass, against 3–12% for vanilla flow matching.

Table 8: Physiological diagnostics on the TUSZ test split (**bold:** best, underline: second best).

Method	TS-FID ↓	PSD slope ↓	Envelope ↓	Hj. act. ↓	Hj. mob. ↓	Drift ↓	Avg. ↓
EEG-GAN	331.4	1.846	1.216	1.362	0.831	0.712	1.193
Vanilla diffusion	121.6	0.684	1.679	1.652	0.571	0.204	0.958
Vanilla flow matching	137.0	0.199	1.651	1.585	<u>0.391</u>	0.102	0.786
JET	199.1	0.862	0.615	<u>0.506</u>	0.673	0.410	0.613
JET [×]	198.5	0.897	<u>0.518</u>	0.467	0.694	0.282	0.571
MontageDiff	40.6	1.141	0.515	0.587	0.480	0.065	<u>0.557</u>
MontageFlow	<u>44.0</u>	<u>0.221</u>	0.580	0.551	0.302	<u>0.069</u>	0.345

8 RELATED WORKS

EEG models. EEG is routinely used to monitor brain activity and is essential for epilepsy diagnosis (Tatum et al., 2018). Deep EEG models are predominantly discriminative and task-specific. For instance, EEGNet uses a compact CNN for EEG classification (Lawhern et al., 2018), while EEG Conformer combines convolution with self-attention to capture long-range temporal dependencies (Song et al., 2023). Gemein et al. (2020) compare deep and feature-based decoders for distinguishing pathological from normal recordings on TUAB, while Tang et al. (2022) and REST (Afzal et al., 2024) use graph neural networks for seizure detection and classification on TUSZ. Even pre-trained EEG foundation models (Kostas et al., 2021; Yang et al., 2023; Jiang et al., 2024; Wang et al., 2025; El Ouahidi et al., 2025) typically require fine-tuning or linear probing on labelled data for downstream tasks (Jiang et al., 2024; Wang et al., 2025; El Ouahidi et al., 2025). However, achieving expert-level detection requires large expert-annotated datasets (Jing et al., 2023). Annotation typically relies on certified neurologists, whose expertise is in short supply (Shah et al., 2018). Moreover, clinically significant target events are rare: seizures account for less than 0.1% of the TUH EEG archive (Shah et al., 2018). Data access is also constrained by patient privacy: even TUH, the largest public EEG corpus, required HIPAA de-identification before release (Obeid & Picone, 2016). Hand-crafted augmentation can expand training data, but its effectiveness is task-dependent, and some transformations may alter the underlying semantics (Rommel et al., 2022). Per-channel augmentation may also violate montage identities (Eq. 5). Alternatively, synthetic seizure EEG can substitute for real seizures in detector training, yielding comparable performance with lower re-identification risk (Pascual et al., 2021). Despite extensive research on EEG-based predictive modelling, EEG generation remains comparatively underexplored.

Generative modeling. Generative modeling has advanced rapidly, driven largely by vision. GANs enabled high-fidelity, high-resolution image synthesis (Goodfellow et al., 2014; Karras et al., 2019), but adversarial training can be unstable and prone to mode collapse (Arjovsky et al., 2017; Gulrajani et al., 2017). Diffusion models (Sohl-Dickstein et al., 2015; Ho et al., 2020; Song et al., 2021b) use a stable, non-adversarial training objective and have surpassed GANs in image quality (Dhariwal & Nichol, 2021). Closely related to diffusion, flow matching offers an alternative approach to generative modeling by learning a velocity field from noise to data (Lipman et al., 2023; Liu et al., 2023b). With Gaussian noise, the two differ mainly in how they weight noise levels (Kingma & Gao, 2023). Architectures for both diffusion and flow-matching models have evolved from U-Nets (Ronneberger et al., 2015; Ho et al., 2020) to transformers operating on latent patches (Peebles & Xie, 2023) and, more recently, on raw pixels while predicting the clean signal (Li & He, 2026). For time series, GAN- and diffusion-based generators (Yoon et al., 2019; Yuan & Qiao, 2024) use generic inductive biases without encoding domain-specific physical constraints. Constrained generative models handle general or inequality-defined feasible sets by intervening during sampling (Christopher et al., 2024) or by modifying the diffusion process (Lou & Ermon, 2023; Liu et al., 2023a). In contrast, the montage constraint is a linear equality with a fixed, closed-form projector applied during both training and sampling. Among EEG generators, EEG-GAN (Hartmann et al., 2018) uses progressive training with a modified WGAN-GP objective. Beyond GANs, Shu et al. (2025) use a diffusion model to synthesize preictal EEG for seizure prediction. Furthermore, JET (Wang et al., 2026) trains a flow-matching transformer with hand-engineered auxiliary temporal and statistical losses. However, EEG-GAN synthesizes low-fidelity samples, as seen in Table 2, and is slow to train (Table 7). JET needs its auxiliary-loss weights tuned for each dataset. None of them generates physically valid EEG. In contrast, MontageFlow synthesizes high-fidelity EEG that is also physically valid.

9 CONCLUSION

We show that current EEG generators synthesize physically invalid signals. MontageFlow rules out such signals by restricting its output to the subspace of valid signals. MontageFlow further conditions the flow matching decoder on a physiological state vector. With latent conditioning, enforcing validity during training also improves fidelity. Across three publicly available large-scale EEG corpora, MontageFlow improves TS-FID by $1.6\text{--}3.6\times$ over the strongest baseline with perfect physical validity. MontageFlow is applicable to other electrical recordings such as ECG though we restrict the scope of our study to EEG. More broadly, our results suggest that validity and fidelity need not trade off when known physical laws are built into the generator.

LIMITATIONS

Although MontageFlow projects the initial noise at sampling time, it trains on unprojected Gaussian noise. This choice is empirical: unprojected training noise yields better scores on TS-FID and the physiological metrics than projected noise. We offer no theoretical account of why this mismatch between training and sampling benefits generation.

AI USE STATEMENT

In this work, we used generative AI tools for AI-assisted coding, drawing the diagrams and polishing the writing. Additionally, we used generative AI tools for finding related works and writing the latex code. We take responsibility for the final content of this work, including text, claims or artifacts produced with the aid of generative AI.

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Appendix

A METRIC DEFINITIONS

TS-FID, shape correlation and the physiological metrics compare the whole split with one generated window per real window. The montage index uses only the generated windows. Real recordings score 0.

TS-FID. The Fréchet distance between Gaussians fitted to the features $\phi(X)$ of real and generated windows,

$$\text{TS-FID} = \|\mu_r - \mu_g\|_2^2 + \text{Tr}(\Sigma_r + \Sigma_g - 2(\Sigma_r \Sigma_g)^{1/2}),$$

with (μ, Σ) the mean and covariance of the features over all windows of the split. Following Wang et al. (2026), ϕ takes the magnitude of the discrete Fourier transform along time and keeps the lower half of the frequency bins. It average-pools frequency to at most 256 bins and channels to 4 groups, applies $\log(1 + \cdot)$, and standardises within the window.

Montage index. Each constraint in Eqs. 2–3 says that two chains of channels between the same pair of electrodes must sum to the same signal. The *residual* is the difference between the two sums. Within each window, we average its absolute value over time and divide by the root-mean-square of the $m = 8$ channels involved. We average this over windows and both constraints, then divide by $\sqrt{2m/\pi} \approx 2.257$, its value for independent Gaussian channels. So 0 is physically valid and 1 is typical of independent channels.

Shape correlation. We average the 16×16 per-window Pearson correlation matrix of the channels over windows. Shape correlation is the Pearson correlation between the off-diagonal entries of this matrix for generated and for real windows. It asks whether the same channel pairs are coupled.

Physiological metrics. Each metric in Table 8 is a per-window scalar $g(X)$. Its distributions over generated and real windows are compared by the 1-Wasserstein distance, divided by the interquartile range of the real values (lower is better),

$$d(g) = W_1(P_g^{\text{gen}}, P_g^{\text{real}}) / \text{IQR}(P_g^{\text{real}}).$$

PSD slope. The exponent χ of a least-squares fit $\log_{10} S(f) \approx a - \chi \log_{10} f$ to each channel’s Welch spectrum over 1–40 Hz, averaged over channels. A signal that is too smooth has a steeper slope.

Envelope. The $\log_{10}$ RMS over time of $e(t) = (\frac{1}{C} \sum_c x_c(t)^2)^{1/2}$. It measures how much energy a window carries.

Hjorth activity. $\frac{1}{C} \sum_c \log_{10} \text{var}(x_c)$, the per-channel counterpart of the envelope.

Hjorth mobility. $\frac{1}{C} \sum_c \sqrt{\text{var}(\dot{x}_c) / \text{var}(x_c)}$, with $\dot{x}_c$ the first difference. It is scale-free, so it reflects frequency content rather than amplitude.

Drift. The least-squares slope of $e(t)$ across the window, multiplied by L and divided by the mean of $e(t)$. It detects windows whose amplitude ramps up or decays.

B QUALITATIVE SAMPLES

Real EEG mixes an ongoing background rhythm with brief sharp events. MontageFlow reproduces the rhythm on all three corpora. On TUSZ its traces are visibly smoother than real. This is the time-domain form of the high-frequency attenuation seen in Figure 4. The baselines fail differently. Vanilla diffusion and vanilla flow matching are far too quiet. JET’s traces are over-regular: they rise and fall with a fixed rhythm.

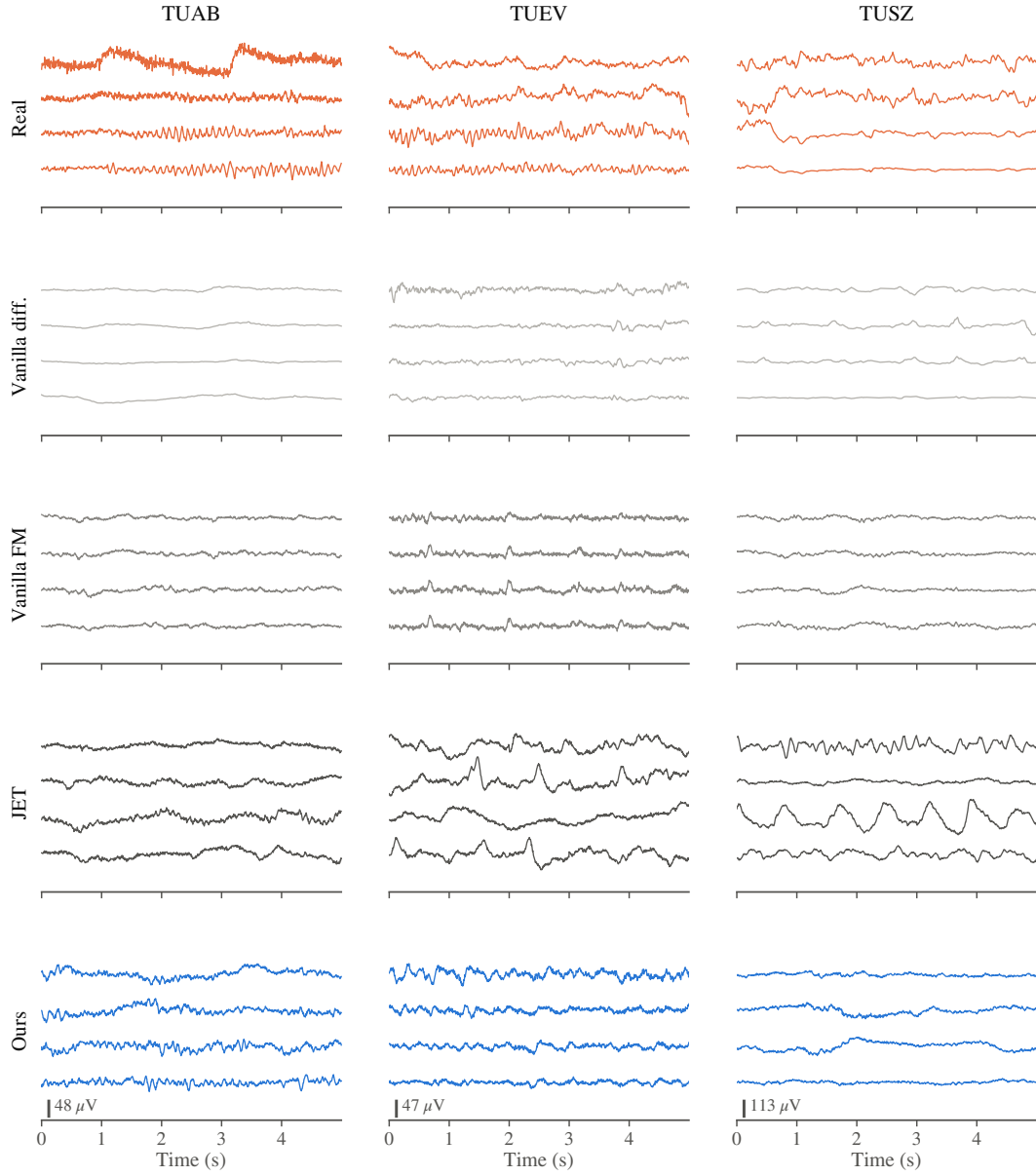


Figure 5: **Generated and real windows.** First four channels of the bipolar montage, five seconds, one window per corpus and method (the median-amplitude window among 2,000 samples).

C CHANNEL-WISE ANALYSIS

Figure 4 averages over the sixteen channels. Figures 6, 7 and 8 show the same three analyses for each channel separately, so an error on a few channels cannot hide in the average. Each curve is a median over the 2,000 windows of that channel.

(1) Power spectrum: Figure 6 shows the power spectrum of each channel. Real EEG has the most alpha power on P3–O1 and P4–O2 and the weakest alpha peak on Fp1–F7, Fp2–F8, Fp1–F3 and Fp2–F4. MontageFlow shows the same pattern on all three corpora. Vanilla flow matching has less power at almost every frequency, and on TUEV it exaggerates the alpha peak on every channel. Above 12 Hz, our spectrum generally falls below the real one but stays within the real shading.

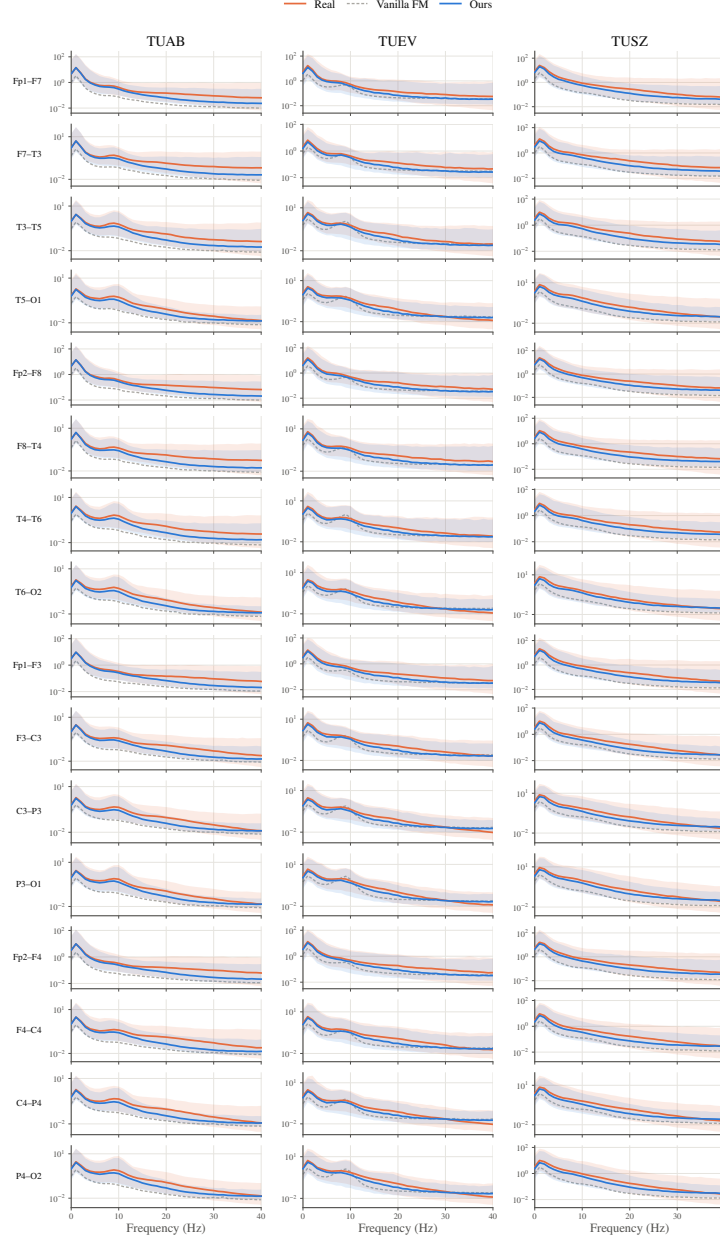


Figure 6: **Channel-wise power spectrum.** Median over windows for each channel, with shading from the 10th to the 90th percentile.

(2) Amplitude over time: Figure 7 tracks the amplitude of each channel across the window. Every envelope is flat, for real EEG and for MontageFlow, so no channel drifts. Real EEG has the largest amplitude on Fp1–F7, Fp2–F8, Fp1–F3 and Fp2–F4 and smallest on T6–O2, C3–P3 and C4–P4. MontageFlow ranks the channels the same way on all three corpora, at 81–95% of the real level. Vanilla flow matching stays at 44–72%.

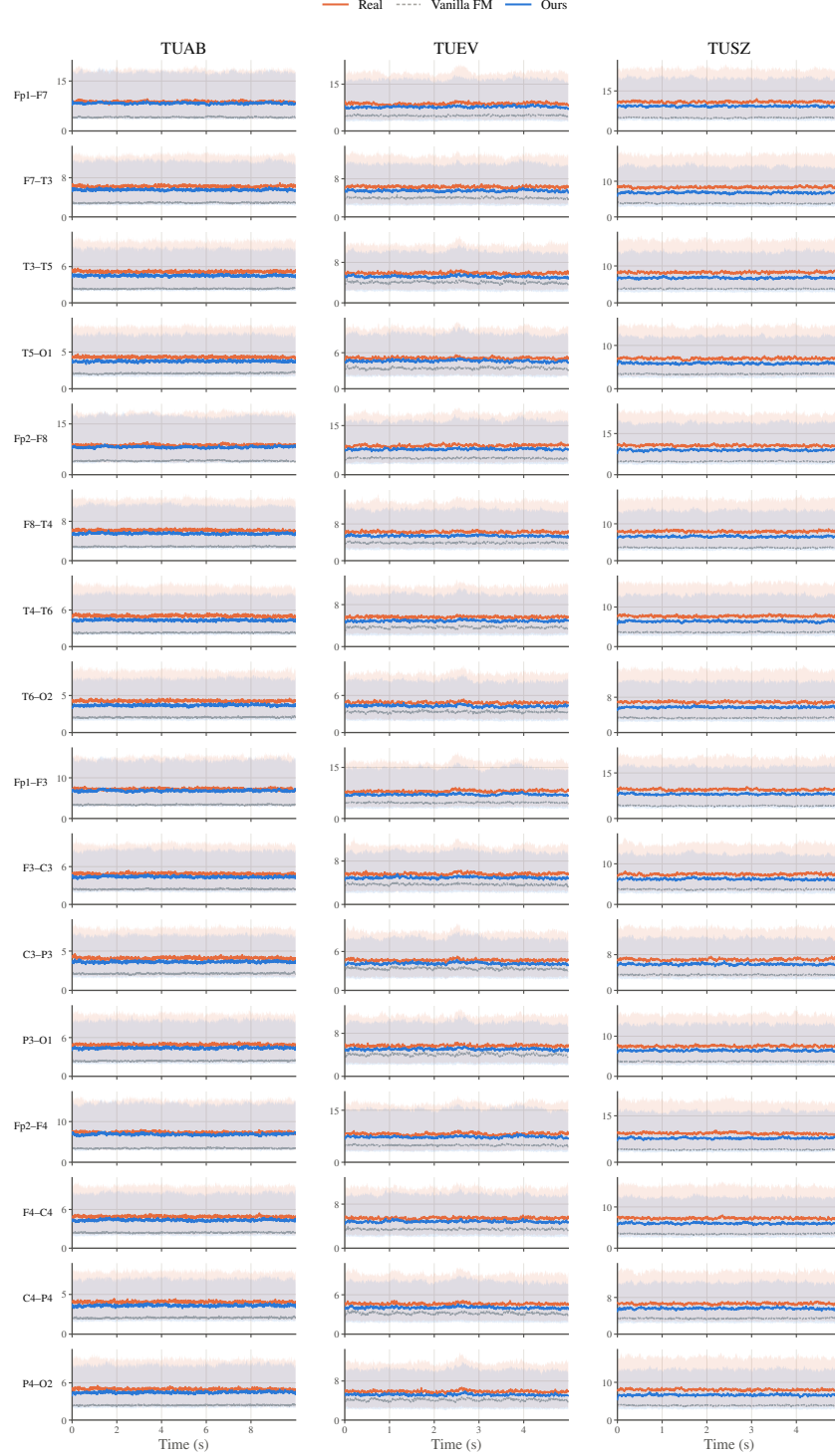


Figure 7: **Channel-wise amplitude over time.** Absolute amplitude of each channel, median over windows, with shading over the middle half of windows.

(3) Heavy-tailed distributions: Figure 8 shows the amplitude distribution of each channel. Our density follows the real curve from the sharp central peak down through the tails on every channel. On TUAB and TUEV, real EEG and MontageFlow both have their heaviest tails on Fp1–F7, Fp2–F8, Fp1–F3 and Fp2–F4. Above $100\ \mu\text{V}$, our samples keep 43–94% of the real probability mass per channel, against at most 18% for vanilla flow matching.

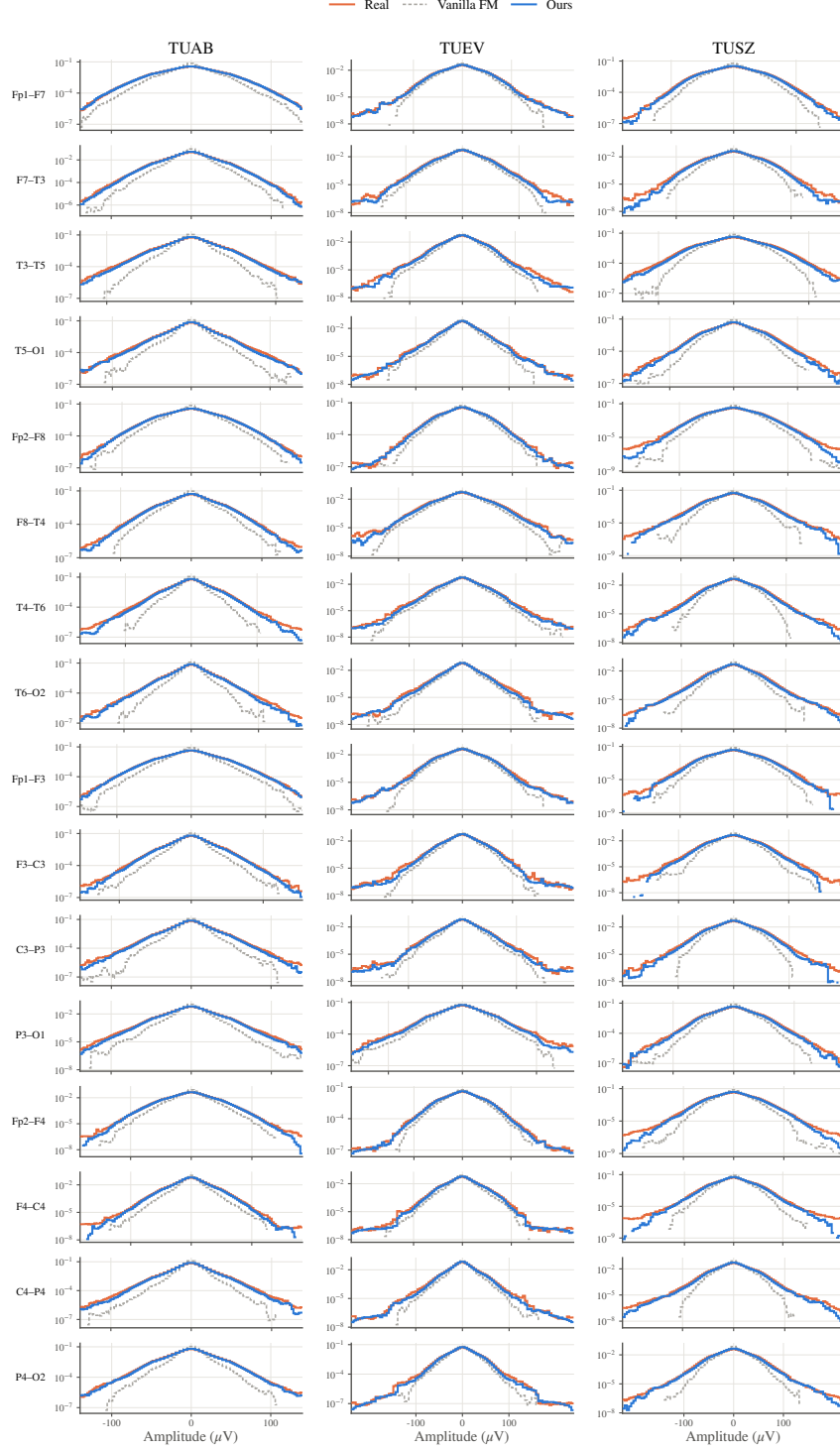


Figure 8: **Channel-wise amplitude distribution.** Distribution of sample amplitudes for each channel (log density).

D POST-HOC PROJECTION ON ALL CORPORA

Table 9 extends Table 6 to all three corpora (test split). Training through the projector wins every cell.

Table 9: Post-hoc projection vs. projection in training on all three corpora, test split (**bold**: best per corpus).

Corpus	Projection	TS-FID ↓	PSD slope ↓	Envelope ↓	Hj. act. ↓	Hj. mob. ↓	Drift ↓
TUSZ	Inference only	63.1	0.257	0.639	0.584	0.471	0.089
	Training + inference	44.0	0.221	0.580	0.551	0.302	0.069
TUEV	Inference only	107.6	0.268	0.267	0.280	0.217	0.129
	Training + inference	93.1	0.230	0.261	0.278	0.210	0.121
TUAB	Inference only	140.6	0.629	0.244	0.256	0.366	0.097
	Training + inference	59.9	0.400	0.164	0.215	0.232	0.092

E MONTAGEDIFF

MontageDiff applies our two components, montage projection and latent conditioning, to a denoising diffusion model (Ho et al., 2020) instead of flow matching. We use the standard linear noise schedule with $T = 1000$ steps, where $\bar{\alpha}_t$ is the fraction of signal variance kept at step t . The network ϵ_θ predicts the noise that was added to a clean window. As in MontageFlow, we project this prediction with P (Eq. 7) and train with an ℓ_1 loss:

$$\mathcal{L}_{\text{MD}} = \mathbb{E}_{X,y,E,t,\eta,\sigma} [\|E - P\epsilon_\theta(X_t, t, y, \tilde{h})\|_1], \quad X_t = \sqrt{\bar{\alpha}_t} X + \sqrt{1 - \bar{\alpha}_t} E. \quad (13)$$

To generate a window, we start from $\hat{X}_T = P\varepsilon$ and take 50 deterministic DDIM steps (Song et al., 2021a). Each step goes from t to the next timestep $s < t$:

$$\hat{X}_s = \sqrt{\bar{\alpha}_s} \frac{\hat{X}_t - \sqrt{1 - \bar{\alpha}_t} \hat{E}_t}{\sqrt{\bar{\alpha}_t}} + \sqrt{1 - \bar{\alpha}_s} \hat{E}_t, \quad \hat{E}_t = P\epsilon_\theta(\hat{X}_t, t, y, h). \quad (14)$$

This is a linear combination of $\hat{X}_t$ and $\hat{E}_t$, which both lie in $\ker A$, so the sample never leaves $\ker A$ and the final window is valid by construction. Everything else is shared with MontageFlow: the backbone, the encoder, the latent prior and the training settings.