

MORE THAN 83.69% OF THE ZEROS OF THE RIEMANN ZETA FUNCTION ARE DISTINCT

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ABSTRACT. The lower asymptotic proportion of distinct nontrivial zeros of the Riemann zeta function, counted with multiplicity, is at least $0.8369928814\dots$. The previous bound was $0.83625\dots$. As in the proof of that bound, an unconditional version of Montgomery’s pair-correlation theorem gives an asymptotic energy estimate. The new ingredient is a short matrix inequality with a free clipping parameter. It strengthens the lower bound for this energy in terms of the number of distinct zeros. The gain is a nonnegative correction from overlaps between different nearby zeros on the critical line, which is retained even when some of these zeros are double. The matrix inequality, the threshold lemma, the block dichotomy, the counting assembly and the exact arithmetic are proved formally in Lean 4. The constant relies on a computer-assisted local inequality from recent work that has not yet been refereed. That computation was re-run independently, and every imported input is listed. This paper is primarily an experiment in AI-assisted mathematical research (Section 4).

1. INTRODUCTION

The Riemann zeta function has infinitely many nontrivial zeros $\rho = \sigma + i\gamma$ with $0 < \sigma < 1$. It is conjectured that every zero is *simple*, meaning not repeated. Let $N(T)$ count the zeros with $0 < \gamma \leq T$, with multiplicity, and let $N_d(T)$ count the *distinct* ones. If all zeros were simple, then $N_d = N$.

Montgomery’s pair-correlation method [8] gives lower bounds for such proportions. An argument discovered by Claude (Anthropic) [4] and verified by Alpöge and Furman [1], together with a second proof by Lamzouri [6], shows unconditionally that $\liminf N_d(T)/N(T) \geq 0.8362503518\dots$. Wang [12] improved this by about $3 \cdot 10^{-8}$. The purpose of this paper is to prove the following bound.

Theorem 1. $\liminf_{T \rightarrow \infty} \frac{N_d(T)}{N(T)} \geq q := \frac{16260119298029}{19426831050000} = 0.83699288145242\dots$

The idea. Rescale the zeros so that their mean spacing is one: a zero $\rho = \frac{1}{2} + i\gamma$ on the critical line gets the coordinate $y_\rho = \gamma \log T / (2\pi)$. Fix an even probability density $f = \eta^2$ on $[-\frac{1}{2}, \frac{1}{2}]$, and attach to this zero the wave packet

$$F_{y_\rho}(u) = \eta(u) e^{2\pi i u y_\rho} \in L^2(-\tfrac{1}{2}, \tfrac{1}{2}).$$

These are unit vectors, and $\langle F_y, F_{y'} \rangle = \hat{f}(y - y')$, where $\hat{f}(x) = \int f(u) e^{-2\pi i x u} du$. The overlap is 1 for coincident zeros, close to 1 for nearby zeros, and small for zeros far apart. Section 3 uses a smoothed version of these vectors, labelled there by $z_\rho = -y_\rho$.

Zeros off the critical line come in pairs placed symmetrically about the line. Each pair contributes a Hermitian term with at most one positive eigenvalue, so the construction needs no assumption about where the zeros lie. Summing the projections onto the vectors above and these pair terms, with multiplicity, gives an operator A with $\text{tr } A = N$. The energy $E = \text{tr } A^2$ is a pair sum over all zeros. For ordered pairs (ρ, ρ') on the critical line, including $\rho = \rho'$,

the summands are $\hat{f}(y_\rho - y_{\rho'})^2$, counted with multiplicity. The full identity, including off-line zeros, is given in Section 3. Montgomery's pair-correlation theorem, in its unconditional form [3], computes this energy asymptotically: for the smoothed operator, $E = (C_\varepsilon + o(1))N$ with $C_\varepsilon \rightarrow C = \int f^2 + \iint |s - t|f(s)f(t) ds dt$, and $C \leq 2 - H < 2$ for the window used here (Proposition 5).

A zero on the critical line of multiplicity ν contributes ν to the trace and ν^2 to the diagonal part of the energy sum. A double zero contributes 4 to this diagonal part, whereas two distinct simple zeros contribute 2, with an additional mutual term $2\hat{f}(y - y')^2$ that is small when they are far apart. Both configurations contribute 2 to the trace. Linear algebra turns this into $E \geq 3N - 2N_d$, hence $\liminf_{T \rightarrow \infty} N_d/N \geq \frac{3-C}{2}$. The Montgomery–Taylor window gives the previous bound this way. The window used here gives up about $2 \cdot 10^{-5}$ of that baseline in exchange for a larger gain below.

The improvement adds a nonnegative *defect* term to this inequality. The defect measures how far the Gram matrix $U = (\hat{f}(y_i - y_j))$ of the distinct on-line zeros of multiplicity one or two is from the identity, and nearby zeros make it positive. The total spread of these points is at most $N + o(N)$. A certified local inequality (Proposition 6) relates the spread to the squared overlaps: for any seven consecutive points, a weighted sum of their squared overlaps plus a multiple of their spread is at least a fixed positive constant. As a result, the defect is at least a fixed amount per point, minus a penalty proportional to how widely the points are spread.

Recent simple-zero records [2, 11, 9, 10] use such a defect for simple zeros. Counting distinct zeros requires keeping the defect when double zeros are present too, and Theorem 2 does exactly that.

2. THE MATRIX INEQUALITIES

For Hermitian X and real f write $\text{tr } f(X) = \sum_i f(\lambda_i(X))$. Write $X \preceq Y$ when $Y - X$ is positive semidefinite. For real c and $\tau \geq 0$ let

$$\varphi_c(x) = \begin{cases} x^2, & x \leq c, \\ 2cx - c^2, & x > c, \end{cases} \quad \Psi_\tau(x) = \begin{cases} (x-1)^2, & x \leq 1 + \tau, \\ 2\tau(x-1) - \tau^2, & x > 1 + \tau. \end{cases}$$

Both functions are C^1 with nondecreasing derivative, hence convex on $\mathbb{R}$.

Theorem 2 (Mixed-multiplicity Gram inequality). *Let U be a Hermitian $l \times l$ matrix with unit diagonal, let $D = \text{diag}(d_i)$ with $1 \leq d_i \leq 2$, and let $G = D^{1/2}UD^{1/2}$. If $\tau \geq 0$ and $c \geq \max(1 + \tau, 2 + \tau/2)$, then*

$$\text{tr } \varphi_c(G) - \text{tr } D^2 \geq \text{tr } \Psi_\tau(U).$$

Lemma 3. *If G, B are Hermitian and $B \preceq 2cI$, then $\text{tr } \varphi_c(G) \geq \text{tr}(BG) - \frac{1}{4} \text{tr } B^2$.*

In control-theoretic terms, Lemma 3 is a dual certificate. Every matrix B that satisfies the linear matrix inequality $B \preceq 2cI$ yields a lower bound on $\text{tr } \varphi_c(G)$, and the bound is attained at $B = 2 \min(G, cI)$. The proof of Theorem 2 exhibits one admissible B .

Proof of Lemma 3. In an eigenbasis of G , $\text{tr}(BG) = \sum b_{ii}\lambda_i$ and $\text{tr } B^2 \geq \sum b_{ii}^2$, with $b_{ii} \leq 2c$. It suffices to show $b\lambda - \frac{1}{4}b^2 = \lambda^2 - (\lambda - \frac{b}{2})^2 \leq \varphi_c(\lambda)$ for $b \leq 2c$. If $\lambda \leq c$ this is clear. If $\lambda > c$, then $\lambda - \frac{b}{2} \geq \lambda - c > 0$, and the claim follows. $\square$

Proof of Theorem 2. Let $\mathcal{C}$ be $U - I$ with each eigenvalue μ replaced by $\min(\mu, \tau)$, so that $\mathcal{C} \preceq \tau I$. Put $M = D^{-1/2}\mathcal{C}D^{-1/2}$ and $B = 2D + 2M$. Then $B \preceq 2D + 2\tau D^{-1} \preceq 2cI$. For the last step,

$d + \tau/d$ is convex on $[1, 2]$, so it is at most $\max(1 + \tau, 2 + \tau/2) \leq c$. Using $U_{ii} = 1$ and cyclicity, $\text{tr}(DG) = \text{tr} D^2$, $\text{tr}(MG) = \text{tr}(\mathcal{C}U)$ and $\text{tr}(DM) = \text{tr} \mathcal{C}$. Hence

$$\text{tr}(BG) - \frac{1}{4} \text{tr} B^2 - \text{tr} D^2 = 2 \text{tr}(\mathcal{C}(U - I)) - \text{tr} M^2.$$

Moreover $\text{tr} M^2 = \sum |\mathcal{C}_{ij}|^2 / (d_i d_j) \leq \text{tr} \mathcal{C}^2$. By Lemma 3,

$$\text{tr} \varphi_c(G) - \text{tr} D^2 \geq 2 \text{tr}(\mathcal{C}(U - I)) - \text{tr} \mathcal{C}^2 = \sum_{\mu} (2\mu \min(\mu, \tau) - \min(\mu, \tau)^2),$$

and each summand equals $\Psi_{\tau}(1 + \mu)$. $\square$

The counting argument also requires a threshold inequality of the kind used in [4, 1]. The following proof avoids eigenvalue interlacing. For Hermitian X let $X_{\pm} \succeq 0$ carry its positive and negative eigenvalues, so that $X = X_+ - X_-$ and $X_+ X_- = 0$.

Lemma 4. *Let $P \succeq 0$, let Q be Hermitian with at most b positive eigenvalues, let $A = P + Q$, and let c be real. Then $\text{tr} A^2 \geq 2c \text{tr} A - 2c \text{tr} P + \text{tr} \varphi_c(P) - c^2 b$.*

Proof. Write $Z = Q_-$, so that $A = (P - Z) + Q_+$. The cross term is $2 \text{tr}((P - Z)Q_+) = 2 \text{tr}(PQ_+) \geq 0$, since $ZQ_+ = 0$. Hence $\text{tr} A^2 \geq \text{tr}(P - Z)^2 + \text{tr} Q_+^2$. Also $\text{tr} Q_+^2 \geq 2c \text{tr} Q_+ - c^2 b$, because $x^2 \geq 2cx - c^2$. It remains to show $\text{tr}(P - Z)^2 + 2c \text{tr} Z \geq \text{tr} \varphi_c(P)$. Since $\varphi_c(p) = p^2 - (p - c)_+^2$, this is $\text{tr} X_+^2 - 2 \text{tr}(XZ) + \text{tr} Z^2 \geq 0$ with $X = P - cI$. Now $\text{tr}(XZ) \leq \text{tr}(X_+ Z)$ because $\text{tr}(X_- Z) \geq 0$. So the left side is at least $\text{tr}(X_+ - Z)^2 \geq 0$. $\square$

3. PROOF OF THEOREM 1

The window. Following [11] (files `design.py` and `kernel.py`), put

$$v(u) = \cos(\sqrt{2}u) + 10^{-9} \sum_{j=1}^6 \kappa_j \cos(2\pi j u) \quad (|u| \leq \tfrac{1}{2}), \quad f = v / \int_{-1/2}^{1/2} v,$$

with $(\kappa_j) = (3322500, -7609135, 1190194, -731476, -1680572, 1141360)$. Both v and f vanish outside $I = [-\frac{1}{2}, \frac{1}{2}]$. On I , $v > 0$ (its minimum there is about 0.75), and $\int v = \sqrt{2} \sin(1/\sqrt{2})$. Let $K(x) = \hat{f}(x) = \int f(u) e^{-2\pi i x u} du$, so that $K(0) = 1$ and $K^2 = (\hat{v}/\hat{v}(0))^2$ is the verifier's kernel.

The operator. Let $L = \log T$ and $z_{\rho} = i(\rho - \frac{1}{2})L/(2\pi)$ for $0 < \gamma \leq T$. On-line zeros give real z_{ρ} . An off-line zero pairs with $1 - \bar{\rho}$, which has the same multiplicity and the point $\bar{z}_{\rho}$.

The function f jumps at $\pm \frac{1}{2}$, so $f * f$ is not twice differentiable, and the smoothing below is necessary. Since $\sqrt{f}$ is smooth and positive on the interior of I , multiplying it by even smooth cutoffs that tend to $\mathbf{1}_I$ and renormalising gives $f_{\varepsilon} = \eta_{\varepsilon}^2$. Here η_{ε} is real, even, smooth and supported inside I , $\int f_{\varepsilon} = 1$, $d_{\varepsilon} = \|f_{\varepsilon} - f\|_1 \rightarrow 0$ and $f_{\varepsilon} \rightarrow f$ in L^2 .

Put $K_{\varepsilon} = \hat{f}_{\varepsilon}$ and $F_z(u) = \eta_{\varepsilon}(u) e^{-2\pi i u z}$. With inner products conjugate-linear in the first slot, $\langle F_z, F_w \rangle = K_{\varepsilon}(w - \bar{z})$. The operator $A = \sum_z \nu_z F_z \langle F_{\bar{z}}, \cdot \rangle$, where ν_z is the multiplicity, is Hermitian. It satisfies $\text{tr} A = N$ and $\text{tr} A^2 = \sum_{z,w} \nu_z \nu_w K_{\varepsilon}(z - w)^2 =: E_{\varepsilon}$, using that K_{ε} is even. All operators act on the finite-dimensional span of the F_z .

Proposition 5 (analytic input [3, Lemma 5]; computer-assisted bound [11]). $E_{\varepsilon} = (C(f_{\varepsilon}) + o(1))N$, where $C(g) = \int g^2 + \iint |s - t|g(s)g(t)$. Moreover $C(f_{\varepsilon}) \rightarrow C(f) \leq 2 - H$ with $H = \frac{3362285207}{5000000000}$.

The asymptotic is the unconditional Montgomery theorem [3, Lemma 5] (see also [6, 1]). It applies to real, even, integrable test functions supported in $[-1, 1]$ and Lipschitz at 0, and it sums

over all zeros, on or off the line, with multiplicity. It carries the weight $w(\rho - \rho') = 4/(4 - (\rho - \rho')^2)$. With $R_\varepsilon = f_\varepsilon * f_\varepsilon$,

$$(R_\varepsilon - R_\varepsilon''/(4L^2))^\wedge(x) = \left(1 + \frac{\pi^2 x^2}{L^2}\right) K_\varepsilon(x)^2.$$

At $x = z_\rho - z_{\rho'}$ the factor is $1 - (\rho - \rho')^2/4 = 1/w(\rho - \rho')$. Both R_ε and R_ε'' are admissible fixed test functions. So applying the lemma to both removes the weight exactly. The R_ε'' term contributes $O_\varepsilon(N/L^2)$, and the total error is $o_\varepsilon(N)$.

The bound $C(f) \leq 2 - H$ is a certified interval enclosure [11]. An independent 40-digit (non-interval) evaluation gives $2 - C(f) = 0.67245704141454428878\dots$, consistent with it.

Splitting. Let n_1, n_2 count the distinct on-line zeros of multiplicity 1, 2. Let h count those of multiplicity ≥ 3 , and let k count off-line pairs. Then $N_d = n_1 + n_2 + h + 2k$. Let P be the part of A coming from the $l = n_1 + n_2$ low-multiplicity points y_j . Then $P = VDV^*$ with $Ve_j = F_{y_j}$, and $U = V^*V$ has $U_{ij} = K_\varepsilon(y_i - y_j)$. So P has the nonzero spectrum of $G = D^{1/2}UD^{1/2}$. Since $\varphi_c(0) = 0$, it follows that $\text{tr } \varphi_c(P) = \text{tr } \varphi_c(G)$. Also $\text{tr } P = n_1 + 2n_2$.

In $Q = A - P$, each high point gives a positive rank-one term. Each off-line pair gives $\nu(vw^* + wv^*) = \frac{\nu}{2}((v+w)(v+w)^* - (v-w)(v-w)^*)$, with $v = F_z$ and $w = F_{\bar{z}}$, which has at most one positive eigenvalue. Since $n_+(\sum_j Q_j) \leq \sum_j n_+(Q_j)$, Q has at most $h + k$ positive eigenvalues.

Fix $\tau \geq 0$ and $c \geq \max(1 + \tau, 2 + \tau/2)$, so that $c \geq 2$. Lemma 4 with $b = h + k$ and Theorem 2 give

$$E_\varepsilon \geq 2cN - (2c - 1)n_1 - (4c - 4)n_2 - c^2(h + k) + \text{tr } \Psi_\tau(U).$$

Compare this with $3N - 2N_d$. A high point of multiplicity $j \geq 3$ leaves $(2c - 3)j + 2 - c^2 \geq r_h := 6c - 7 - c^2$, and an off-line pair of multiplicity $j \geq 1$ leaves $(4c - 6)j + 4 - c^2 \geq r_k := 4c - 2 - c^2$. Hence

$$(1) \quad E_\varepsilon \geq 3N - 2N_d + \text{tr } \Psi_\tau(U) + r_h h + r_k k.$$

The local defect.

Proposition 6 (imported, computer-assisted [11]). *There are rational $w_{ij} \geq 0$ with $\sum_i w_{i,i+r} = 2$ for $r = 1, \dots, 6$, such that for all real $y_0 \leq \dots \leq y_6$,*

$$\frac{1}{2736}(y_6 - y_0) + \sum_{0 \leq i < j \leq 6} w_{ij} K(y_j - y_i)^2 \geq \delta := \frac{891}{200000}.$$

Take the Gram matrix U_B , built with K , of m consecutive points. It is positive semidefinite because $f \geq 0$. Summing Proposition 6 over the $m - 6$ windows gives weight at most 2 per pair. Since $\text{tr}(U_B - I)^2 = 2 \sum_{i < j} K(y_j - y_i)^2$, this gives $\text{tr}(U_B - I)^2 + \frac{1}{2736}W \geq \Delta_m := (m - 6)\delta$, where W is the total window length. If $\Delta_m \leq \tau^2$, then $\text{tr } \Psi_\tau(U_B) + \frac{1}{2736}W \geq \Delta_m$. Indeed, if every eigenvalue is at most $1 + \tau$, then $\text{tr } \Psi_\tau(U_B) = \text{tr}(U_B - I)^2$. Otherwise one eigenvalue alone contributes at least τ^2 , and $\Psi_\tau \geq 0$.

Replacing K by K_ε changes each entry by at most d_ε , so the block changes by at most md_ε in norm. Both blocks are Gram matrices, so their eigenvalues lie in $[0, \infty)$, where Ψ_τ is $\max(2, 2\tau)$ -Lipschitz. By Weyl's inequality, $\text{tr } \Psi_\tau$ changes by at most $\max(2, 2\tau)m^2 d_\varepsilon$.

By convexity of Ψ_τ (the pinching inequality $\text{tr } \Psi_\tau(X) \geq \sum_B \text{tr } \Psi_\tau(X_{BB})$), $\text{tr } \Psi_\tau(U)$ dominates the sum over any partition into consecutive blocks. Average over the m shifted partitions:

- the full blocks over all shifts number at least $l - 2m$, so the Δ_m terms total at least $\Delta_m(l - 2m)/m$;
- each window lies inside a block for at most $m - 6$ of the shifts, and each gap lies in at most 6 windows, so the W terms total at most $6(m - 6)(y_l - y_1)/(2736m)$.

The result is

$$(2) \quad \text{tr } \Psi_\tau(U) \geq a l - \beta (y_l - y_1) - O_{m,\tau}(d_\varepsilon) N - O_m(1), \quad a = \frac{\Delta_m}{m}, \quad \beta = \frac{6(m-6)}{2736m}.$$

Here the spread $y_l - y_1$ is defined to be 0 when $l \leq 1$. If no shift has a full block, (2) still holds: its right side is then at most 0 once the constant in $O_m(1)$ is taken at least $2\Delta_m$, and $\text{tr } \Psi_\tau(U) \geq 0$. Shifted-block pinching is from [2]. The window-in-block pressure count is from [10]; see also [11, 9]. What is new is the use of both on *all* distinct points of multiplicity one or two.

Constants. Take $\tau = \frac{12}{5}$, $c = \frac{17}{5}$ and $m = 1298$. Then $\Delta_m = \frac{287793}{50000} < \tau^2 = \frac{144}{25}$, $a = \frac{26163}{5900000}$ and $\beta = \frac{17}{7788}$. Also $r_h - a = \frac{10829837}{5900000} > 0$ and $r_k - 2a = \frac{91837}{2950000} > 0$.

Conclusion. The points y_j lie in an interval of length $TL/2\pi = N + O(T) = N + o(N)$, and $l = N_d - h - 2k$. So (1), (2) and Proposition 5 give

$$(2 - a)N_d \geq (3 - C(f_\varepsilon) - \beta - O_{m,\tau}(d_\varepsilon))N + (r_h - a)h + (r_k - 2a)k - o(N).$$

Let $T \rightarrow \infty$ and then $\varepsilon \rightarrow 0$. This gives $\liminf N_d/N \geq (1 + H - \beta)/(2 - a) = q$ in exact arithmetic. $\square$

4. VERIFICATION AND NATURE OF THIS WORK

Formal proofs. The following are proved in Lean 4 with Mathlib [7]; the source is in the ancillary files:

- Lemma 3;
- Theorem 2, for all $\tau \geq 0$ and $c \geq \max(1 + \tau, 2 + \tau/2)$;
- Lemma 4, in both exact-count and “at most b ” forms;
- the block dichotomy;
- the combination of Lemma 4 and Theorem 2 (the inequality preceding (1));
- the counting assembly leading to $(2 - a)N_d \geq (1 + H - \beta)N$ – errors, from explicitly stated hypotheses;
- all exact constants.

Every theorem depends only on the axioms `propext`, `Classical.choice` and `Quot.sound`. The code contains no `sorry`.

Not formalized:

- the operator construction and its reduction to matrices: $\text{tr } P = n_1 + 2n_2$, $\text{tr } \varphi_c(P) = \text{tr } \varphi_c(G)$, and $n_+(Q) \leq h + k$;
- the energy asymptotic;
- the certified local inequality;
- the pinching and block averaging;
- the smoothing $K \rightarrow K_\varepsilon$.

These enter the Lean statements only as explicit hypotheses.

Computation. Proposition 6 comes from the Arb-based [5] verifier of [11], commit 1610b97. Its exhaustive search was re-run on separate hardware: 96 shards and 2,168,370 interval nodes, all verified. Both table hashes and all pruning counts match the published run. The run record is in the ancillary files. The verifier code was not audited line by line. As noted upstream, this certificate has no tangent-free replay yet: 453,136 of its nodes are certified by a convex-tangent pruner. The enclosure of $C(f)$ is taken from [11], with the independent check stated above.

Status. The analytic inputs [1, 6, 3] and the certificates [2, 11, 9, 10] are recent and mostly not yet refereed.

Nature of this work. This paper is primarily an experiment in AI-assisted mathematical research. The arguments, the Lean formalization and much of the text were developed with substantial assistance from AI systems (OpenAI Codex and Anthropic Claude) under the author’s

direction. The author defined and steered the research programme, reviewed all steps, and found the τ -generalisation together with OpenAI Codex. The author takes full responsibility for the content.

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